DISTRIBUTION STATEMENT A
Approved for public release;
Distribution Unlimited

CML-80-1 NASA CR 159884

CONTACT LAW AND IMPACT RESPONSES OF LAMINATED COMPOSITES

C.T. Sun
S.H. Yang

February 1980

COMPOSITE MATERIALS LABORATORY

DTIC
ELECTE
NOV 1 6 1995
S D
F

DTIC QUALITY INSPECTED 5

DEPARTMENT OF DEFENSE
PLASTICS TECHNICAL EVALUATION CENTER
ARRADCOM, DOVER, N. J. 07801

PLASTEC 38535

PURDUE UNIVERSITY
School of Aeronautics and Astronautics
West Lafayette, Indiana 47907

D120113

DTIC
SELECTE
NOV 1 6 1995
S D
F

CML-80-1 NASA CR 159884

CONTACT LAW AND IMPACT RESPONSES OF LAMINATED COMPOSITES

C.T. Sun
S.H. Yang

February 1980

DTIC-AI
Distribution / 11-2-95
Availability Codes
Dist A-1
Avail and / or Special

19951114 098

DISTRIBUTION STATEMENT A
Approved for public release;
Distribution Unlimited

1. Report No. NASA CR 159884	2. Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle Contact Law and Impact Responses of Laminated Composites		5. Report Date February 1980
		6. Performing Organization Code
7. Author(s) C. T. Sun and S. H. Yang		8. Performing Organization Report No.
		10. Work Unit No.
9. Performing Organization Name and Address Purdue University School of Aeronautics and Astronautics West Lafayette, IN 47907		11. Contract or Grant No. NSG3185
		13. Type of Report and Period Covered Interim Report
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington DC 20546		14. Sponsoring Agency Code

15. Supplementary Notes

Project Manager: C. C. Chamis
Structures & Mechanical Technologies Division
NASA-Lewis Research Center
21000 Brookpark Road, Mail Stop 49-3; Cleveland, OH 44135

16. Abstract

Static identation tests were performed to determine the law of contact between a steel ball and glass/epoxy and graphite/epoxy laminated composites. For both composites the power law with an index of 1.5 was found to be adequate for the loading curve. Substantial permanent deformations were noted after the unloading. A high order beam finite element was used to compute the dynamic contact force and response of the laminated composite subjected to the impact of an elastic sphere. This program can be used with either the classical Hertzian contact law or the measured contact law. A simple method has been introduced for estimating the contact force and contact duration in elastic impacts.

17. Key Words (Suggested by Author(s))	18. Distribution Statement
Indentation law, impact response; contact force, contact duration, permanent deformation, finite element, transient response, composite beams, composite plates, structural mechanics	Unclassified, Unlimited

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price*
Unclassified	Unclassified	103	

* For sale by the National Technical Information Service, Springfield, Virginia 22161

NASA-C-168 (Rev. 10-75)

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

Page

NOMENCLATURE

A	Cross-sectional area of beam
A_{ij}, B_{ij}, D_{ij}	Laminate stiffness
D	Depth of beam or bending rigidity of beam
E	Young's Modulus
E_b	Young's Modulus of beam
E_L	Young's Modulus in the fiber direction
E_s	Young's Modulus of isotropic sphere
E_T	Young's Modulus in the transverse direction
F	Contact force
F_{max}	Maximum contact force
G_{LT}	Shear Modulus
I	Moment of inertia
K	Kinetic energy
K_t	Total kinetic energy
K^*	K/F_{max}^2
K_{mn}	Eigen value of the (m,n) mode
L	Span or length of beam
L_1	Linear operator (bending)
L_2	Linear operator (shear)
Q_{mn}	Generalized force
$\overline{Q}_{ij}$	Reduced stiffness of composite material
R_s	Radius of sphere
T	Impact duration
T_n	Period of the nth mode
U	Potential or strain energy
U^*	U/F_{max}^2
$W(x,t)$	Deflection of beam or plate
$W_n(x)$	Eigen function of the nth mode

a	Dimension of plate
a_i	Constant coefficients (i =1,6)
b	Dimension of plate
f_1	Strain energy function for simply-supported beam
f_3	Strain energy function for simply-supported plate
g_1	Kinetic energy function for simply-supported beam
g_3	Kinetic energy function for simply-supported plate
h	Depth of beam or plate
k	Contact Modulus
k^*	Contact force per unit indentation depth
$[k]$	Stiffness matrix
m_s	Mass of sphere
m_t	Mass of target or equivalent mass at time t
$[m]$	Mass matrix
n	Index of indentation power law (loading)
q	Index of indentation power law (unloading)
$q(x,t)$	Forcing function
s	Laplace transformation parameter
t	time
v_s	Velocity of sphere
v_t	Velocity of target
w_b	Bending displacement
$\overline{w}_b$	Laplace transformed function of w_b
w_s	Transverse shear deformation
$\overline{w}_s$	Laplace transformed function of w_s

α	Indentation depth
α_o	Permanent indentation
α_m	Maximum indentation
β	α/R_s, nondimensional indentation
ρ	Mass density of beam
κ	Curvature, or shear correction factor
ξ	1/reduced mass
η	α/α_{max}, relative indentation
ω_n	Natural frequency of the nth mode of beam
ω_{mn}	Natural frequency of the (m,n) mode of plate
∇^2	Laplacian operator
ψ_x, ψ_y	Rotations of plane sections of plate

1. Introduction

It has been a known fact that laminated fiber composites currently in use are relatively weak in resisting impact loads. Great attention has been given to modeling the dynamic behavior of composites subjected to foreign object impacts and to the search for new forms of composites that are capable of improving the impact-resistant property.

Failure modes in composites resulting from the impacts of a hard object and a soft object are, in general, quite different. If the object is relatively rigid and small, then the contact time is short and extensive damage is usually confined to the neighborhood of the contact region. How to quantify the amount of damage received by the composite in the impact zone becomes the central question in the hard-object impact problem.

There are several major factors which could affect the amount of damage in a laminated composite due to the impact of a hard object. Among them are the mass and approach velocity of the object, the bending rigidity of the laminate, and the contact behavior (or the contact law). Many researchers have correlated the impact velocity with the damage for a given mass. Such relationship between the damage and impact velocity becomes invalid if the mass of the striker or the bending property of the laminate is changed. The use of a single parameter which could account for the combined effect of the above mentioned variables is highly desirable.

Energy dissipation takes place in the process of impact that results in damage. It is thus reasonable to use this amount of energy consumed in the impact zone to measure the degree of damage in the target composite beam. There could be various damage modes such as breakage of

fibers, cracking in the matrix, delamination, and plastic deformations, which could all contribute to the energy dissipation in the impact zone. It is conceivable that analytical estimates of the energies associated with these damage modes are prohibitive. A static indentation test which produces the loading-unloading curve may prove to be a simple means for determining such damage energy, since the energy dissipated during the loading and unloading cycle is simply the area enclosed by the curve.

In this report, results of the indentation tests on glass/epoxy and graphite/epoxy laminated composites are presented. The results show that the loading curve follow the power law with a power index 1.5, which is identical to the classical Hertzian contact law. Substantial permanent deformations are observed even when loaded at very low load levels. The unloading curves also follow a power law.

A high order beam finite element is used for computing the dynamic response of laminated composite beams subjected to the impact of an elastic sphere. This finite element includes the classical elastic Hertzian law of contact as well as the measured contact law. The computer program developed for this beam finite element is listed as Appendix A. A simple method has been developed for computing the contact force and contact duration. An estimate of the contact duration is needed in the finite element program in selecting a proper time increment in the time integration procedure. This method is found to be quite accurate except for very thin beams.

2. Indentation Law for Hard Object Impact of Composites

2.1 Hertzian Law of Contact

When two solid bodies are in contact, deformation takes place in the contact zone and the contact force results. Once the contact force is obtained, conventional methods for stress analysis can be used to find the stress distribution in the bodies. To determine the contact force - indentation relationship often becomes the most important step in analyzing the contact problem.

The most famous elastic contact law was developed by Hertz [1] for the contact of two spheres of elastic isotropic materials. The problem was solved based on the theory of elasticity. A special case is that if the radius of one of the spheres becomes infinite, then the problem becomes the contact of an elastic sphere and an elastic half space. The contact force F and the indentation depth α were found to have the relation

$$F = k\,\alpha^{3/2} \tag{2-1}$$

where

$$k = \frac{4}{3} R_s^{1/2} \left[\frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2} \right]^{-1} \tag{2-2}$$

In Eq. (2-2), R_s is the radius of the sphere, ν is the Poisson's ratio, E is the Young's modulus, and the subscripts 1 and 2 indicate the two bodies. Equation (2-1) is usually called the Hertzian law of contact for a sphere on half space.

The 3/2 power law given by Eq. (2-1) was found to be valid by Willis [2] for a rigid sphere pressed on a transversely isotropic half space. A modified contact law with

$$k = \frac{4}{3} R_s^{1/2} \left[\frac{1-\nu_s^2}{E_s} + \frac{1}{E_T} \right]^{-1} \tag{2-3}$$

was employed by Sun [3] for a study on impact of laminated composites. In Eq. (2-3), R_s, ν_s and E_s are the radius, the Poisson's ratio and the Young's modulus of the isotropic shpere, respectively, and E_T is the Young's modulus of the fiber-reinforced composite normal to the impact plane.

In applying the classical Hertzian contact law to the impact of laminated fibrous composites we face several uncertainties. First, the half space assumption is not valid since the laminates in use are of finite thickness. Second, the anisotropic and nonhomogeneous property of laminated compostes may alter the form of the law. Third, the strain rate effect which is not accounted for by the Hertzian law may have significant effect on the F-α relation. Except for the strain rate effect, the first two uncertainties are solvable by analyzing the exact contact problem of a sphere pressed into a laminated composite using three-dimensional elasticity. However, experience tells us that analytical solutions for such contact problems are extremely difficult to obtain especially if permanent deformations are to be accounted for during unloadings. Since unloading paths are particularly important in our study, the experimental approach is taken to determine the law of contact for composites. However, in this study, the strain rate effect is still neglected.

2.2 Indentation Law for Laminated Composites

2.2.1 Theoretical Model

In this study the general form for the indentation law for laminated composites is extended from the classical Hertzian Law. We assume that for loading

$$F = k\,\alpha^n \tag{2-4}$$

where k and n will be determined experimentally. It is obvious that when $n = 3/2$ and k is given by Eq. (2-2), this relation becomes the Hertzian law for isotropic bodies. It is noted that the constant k has a very strange unit if n is not an integer. Also, the value of k depends on the unit used for α. A more physically meaningful expression may be derived by using a nondimensional indentation depth

$$\beta = \alpha/R_s \tag{2-5}$$

with which the indentation can be written as

$$F = k^*\,\beta^n \tag{2-6}$$

In Eq. (2-6), k* has the unit of force. For the Hertzian law,

$$k^* = \frac{4}{3} R_s^2 \left[\frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2} \right]^{-1} \tag{2-7}$$

Permanent indentations in composite targets are usually generated even at relatively low projectile impact speeds. The permanent indentation accounts for the major part of the energy loss of the projectile. Some energy imparted from the projectile to the target can be stored in the form of vibrational energy in the target. As far as the local damage at the impact zone is concerned, the permanent indentation is of more interest to us. For this reason, the force-indentation law for the recovery process must be established. In this study, we assume, in the recovery process,

$$F = F_m \left[\frac{\alpha - \alpha_o}{\alpha_m - \alpha_o} \right]^q \tag{2-8}$$

where F_m is the maximum contact force just before unloading takes place, α_m is the identation corresponding to F_m, and α_0 is the permanent indentation. This recovery law was proposed by Barnhart and Goldsmith [4] for impact of a steel ball onto an armor plate.

2.2.2 Experimental Results

The experimental set-up is depicted by the sketch in Fig. 2.1. The indentation was measured by a dial gage that permits reading up to 1/5000 in. The dial gage was mounted on the loading piston so that only the relative displacement between the indentor and the beam was recorded. The indentor was a steel ball of $\frac{1}{4}$ in. diameter. The beam was clamped at both ends with various spans.

Two types of laminated composites have been tested, namely glass/epoxy and graphite/epoxy. The glass/epoxy was Scotch Ply 1002 by the 3M Company. It contained 10 0^{o}-plies and 9 90^{o}-plies which alternate in the layup with one 0^{o}-ply on top and one at the bottom. The thickness of the beam was 0.19 in. and the width was 1.5 in. The graphite/epoxy specimens were $[0/(\pm45)_2/0_2/\pm45]_s$ laminates. Three different spans, 2 in., 4 in. and 6 in., were used for the glass/epoxy laminates and two spans, 2 in. and 4 in., were used for the graphite/epoxy laminates.

The Loading Curve

For the glass/epoxy laminate, three sets of loading data were obtained for each span. These data were used to determine the best fit for the power law, Eq. (2-4), using the least squares method. The results were presented in Figs. 2.2-2.4. The power indices for the three cases appear to be rather close to that of the classical Hertzian law for isotropic media,

i.e., $n = 1.5$. The small deviation from $n = 1.5$ could be due to measurement errors. For this reason, we set $n = 1.5$ and then determined k by using the least square fit. The resulting curves are shown in Figs. 2.5-2.7. These curves seem to fit the data very well also.

The results of the indentation test on the graphite/epoxy laminated beams are presented in Figs. 2.8-2.10. For the 2-inch span, the best least square fit is $n = 1.5$; and for the 4-inch span as shown in Fig. 2.10, $n = 1.5$ also yields a very good fit.

Table 1 summarizes the indentation laws (the loading portion) obtained from the experimental results for a glass/epoxy composite and a graphite/epoxy composite. It is interesting to note that with $n = 1.5$, the values of k for different spans are almost a constant. This indicates that the indentation law is independent of span. In other words, the bending stress does not influence the "contact rigidity".

Table 2 presents the indentation laws in terms of β and k^* with $n = 1.5$ (see Eq. (2-6)).

The Unloading Curve

Form the test results we have observed that permanent deformation would occur after an indentation test no matter how small the load was. The unloading paths are very different from the loading path as can be seen from Figs. 2.11-2.14 for both glass/epoxy and graphite/epoxy. The unloading curve is modeled by using Eq. (2-8) in which q and α_0 have to be determined. Since the permanent indentation depth α_0 is difficult to measure, the whole data for each unloading path were taken to determine the two parameters q and α_0. The value $q = 2.5$ seems to yield the best overall fit as shown in Figs. 2.11-2.14. For $q = 3.0$ (see Figs. 2.15-2.18) α_0 becomes negative in some cases.

Table 1. Indentation law $F = k\,\alpha^n$ (α in inches).

		Glass/Epoxy $[(0/90)_4/0/90/0/(90/0)_4]$			Graphite/Epoxy $[0/(\pm45)_2/0_2/\pm45]_s$	
Span		2"	4"	6"	2"	4"
Least Squares Fit	n	1.54	1.54	1.66	1.5	1.63
	k	5.569×10^5	5.603×10^5	9.655×10^5	5.964×10^5	9.99×10^5
1.5 Power Fit	n	1.5	1.5	1.5	1.5	1.5
	k	4.617×10^5	4.633×10^5	4.592×10^5	5.964×10^5	5.126×10^5
Modified Hertzian Law Eq.(2-3)		$F = 5.461 \times 10^5\,\alpha^{1.5}$ $R_s = 0.125"$, $\nu_s = 0.3$, $E_s = 30 \times 10^6$ psi. $E_T = 1.2 \times 10^6$ psi.			$F = 5.24 \times 10^5\,\alpha^{1.5}$ $E_T = 1.15 \times 10^6$ psi.	

Table 2. Indentation law $F = k^* \beta^n$

		Glass/Epoxy $[(0/90)_4/0/90/0/(90/0)_4]$			Graphite/Epoxy $[0/(\pm 45)_2/0_2/\pm 45]_s$	
Span		2"	4"	6"	2"	4"
1.5 Power Fit	n	1.5	1.5	1.5	1.5	1.5
	k*	2.0405×10^4	2.0475×10^4	2.0294×10^4	2.6357×10^4	2.2654×10^4
Modified Hertzian Law Eq. (2-6)		$F = 2.4134 \times 10^4 \beta^{1.5}$			$F = 2.32 \times 10^4 \beta^{1.5}$	

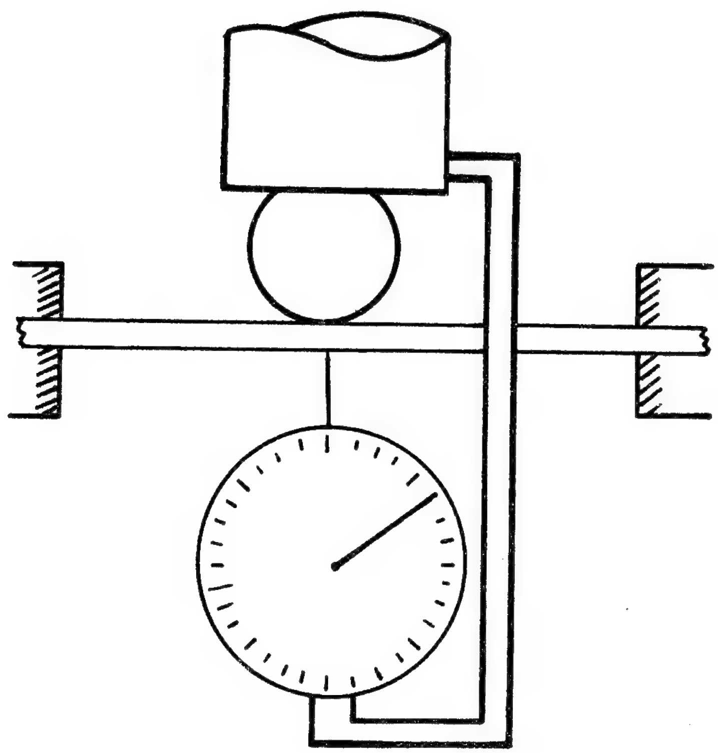

Fig. 2.1 Indentation test setup

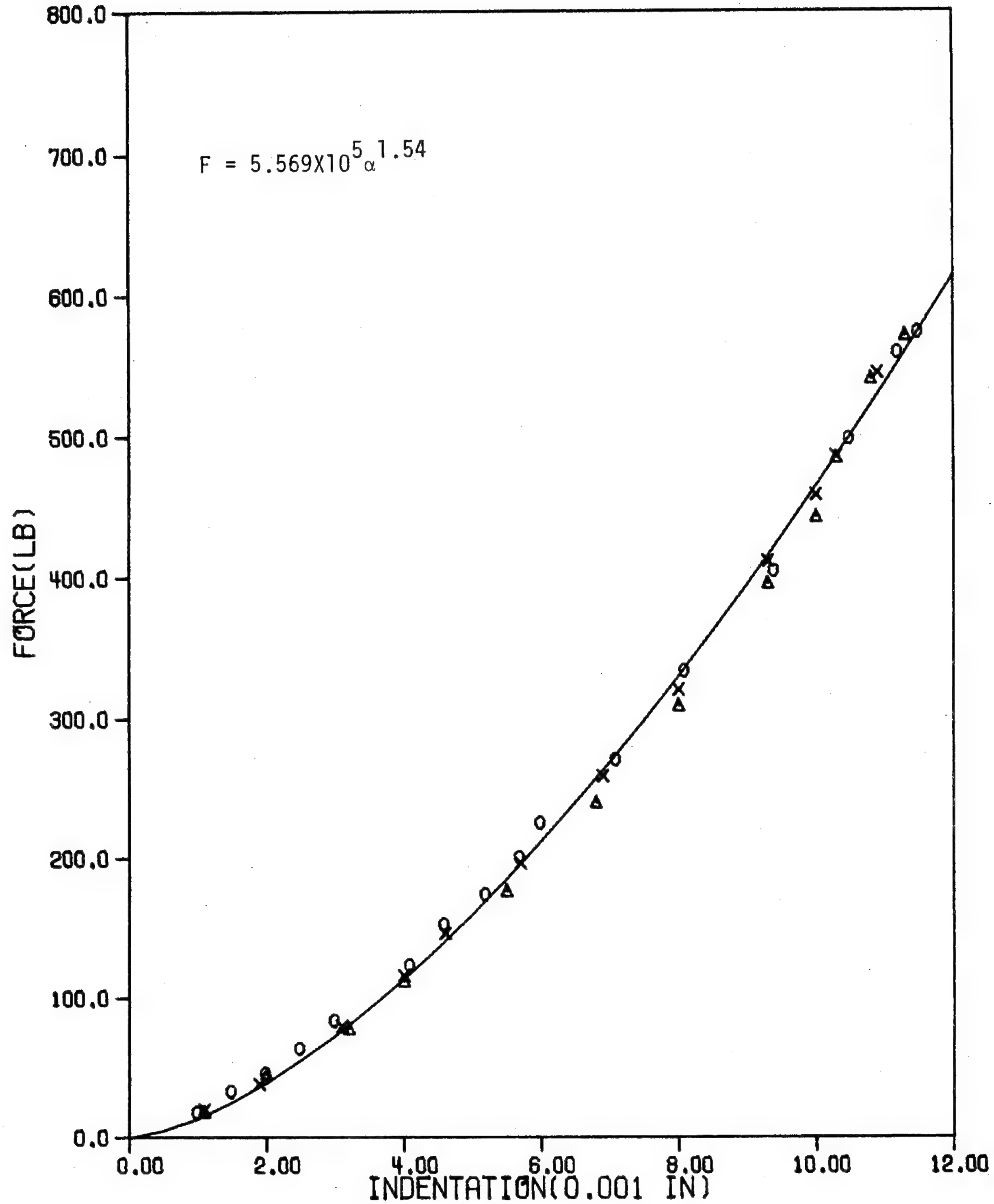

Fig. 2.2. Least-square fit of the contact force - indentation relation for glass/epoxy with 2-inch span.

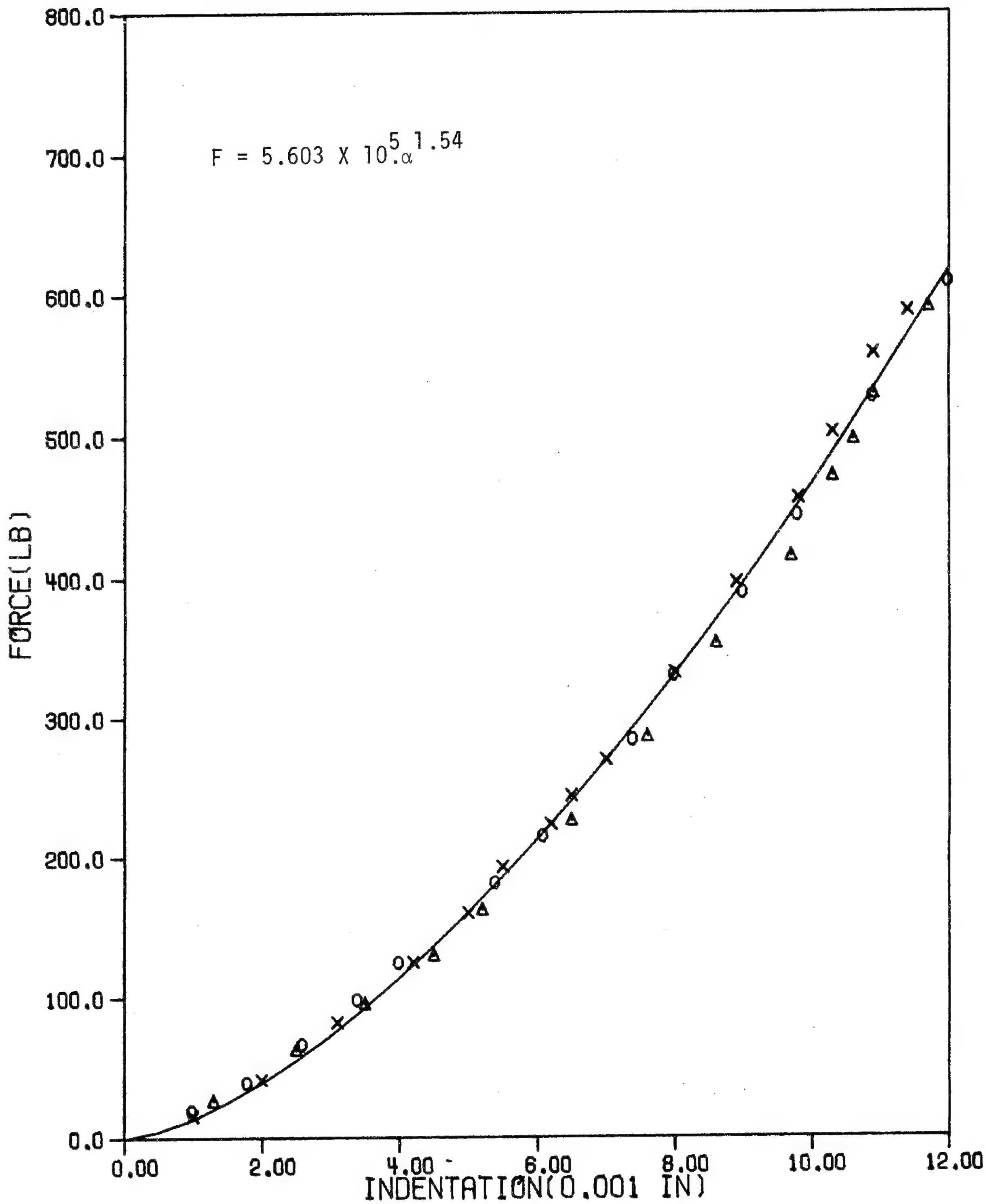

Fig. 2.3. Least-square fit of the contact force-indentation relation for glass/epoxy with 4-inch span.

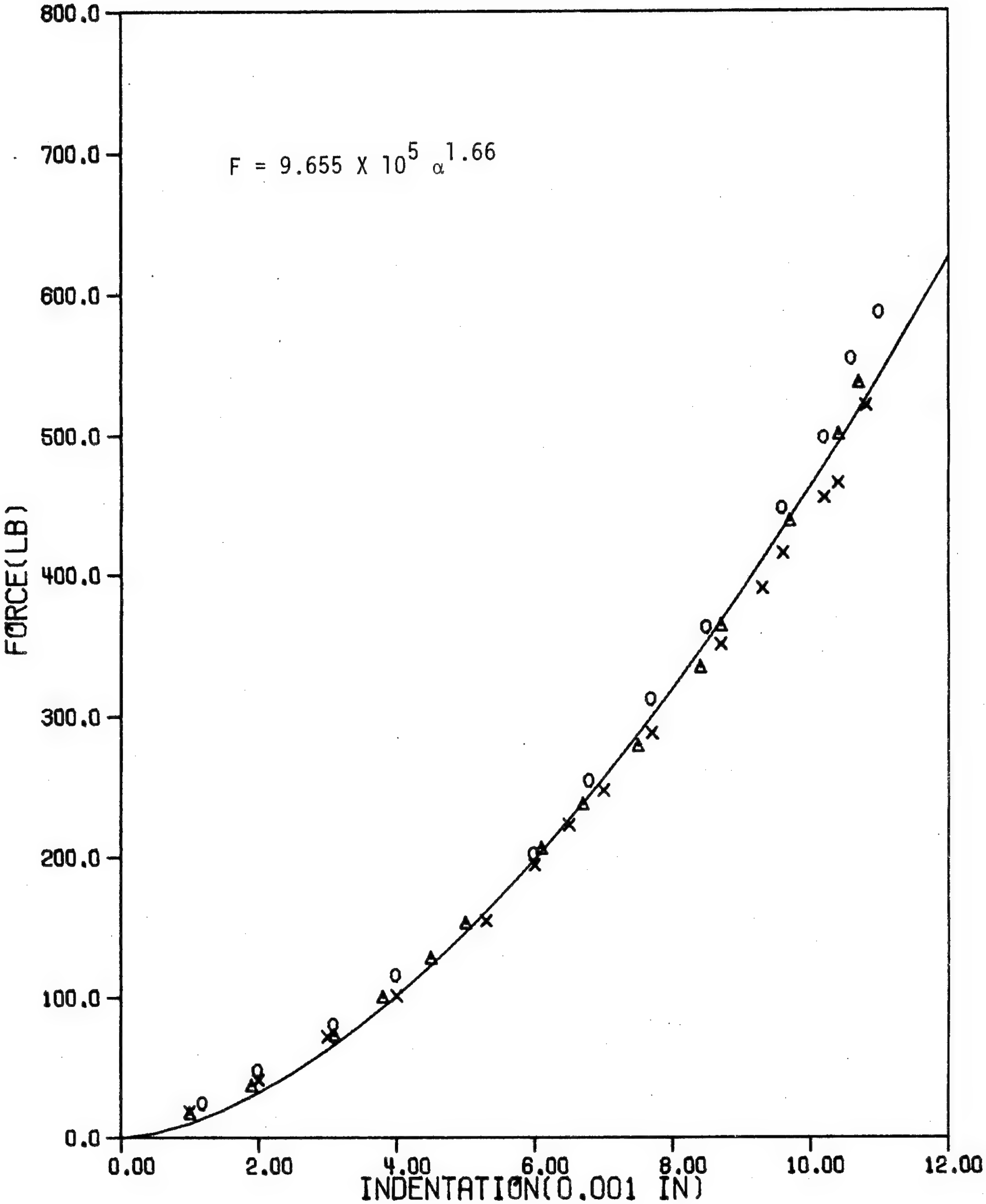

Fig. 2.4. Least-square fit of the contact force - indentation relation for glass/epoxy with 6-inch span.

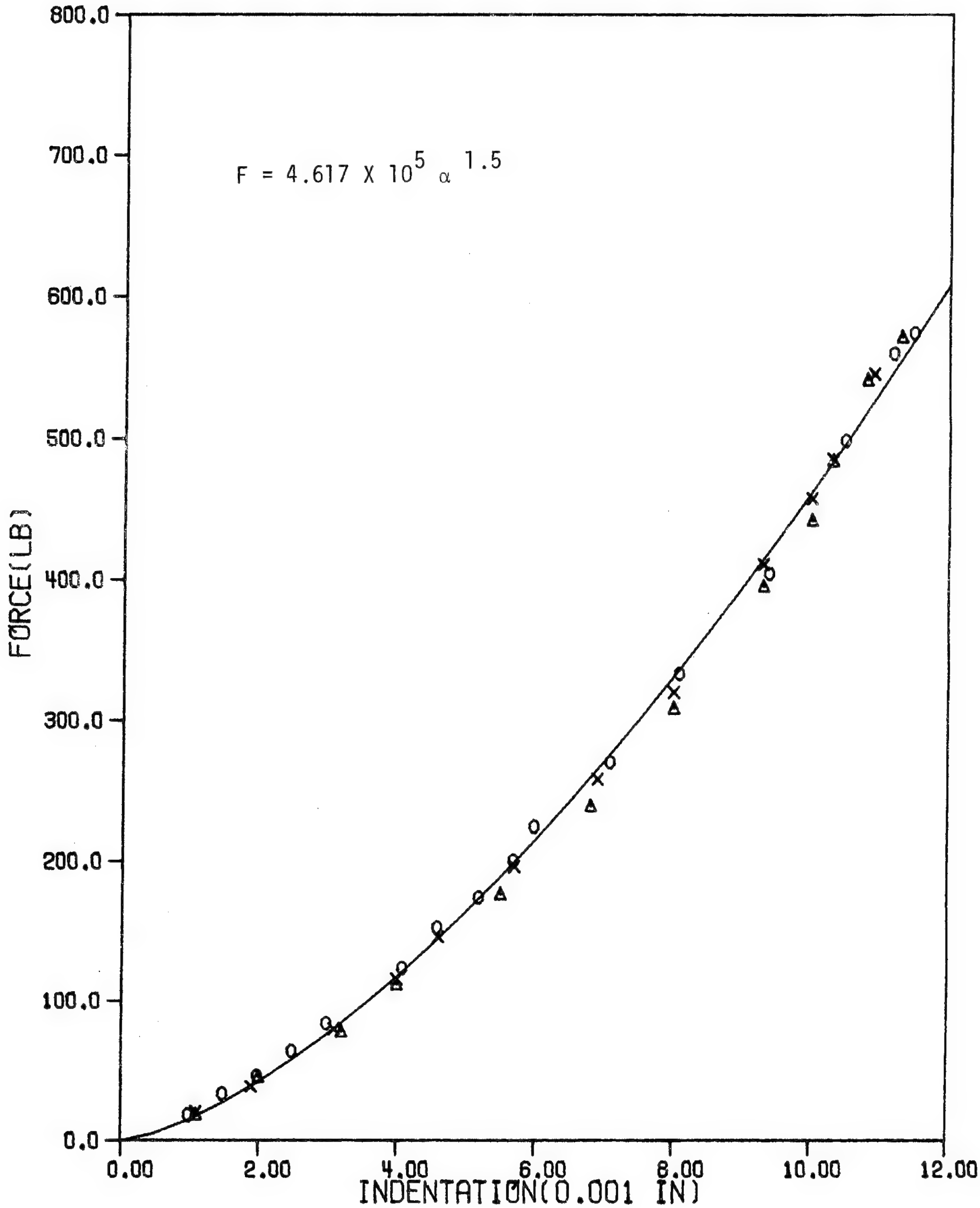

Fig. 2.5. Least-square fit with $n = 1.5$ for glass/epoxy with 2-inch span.

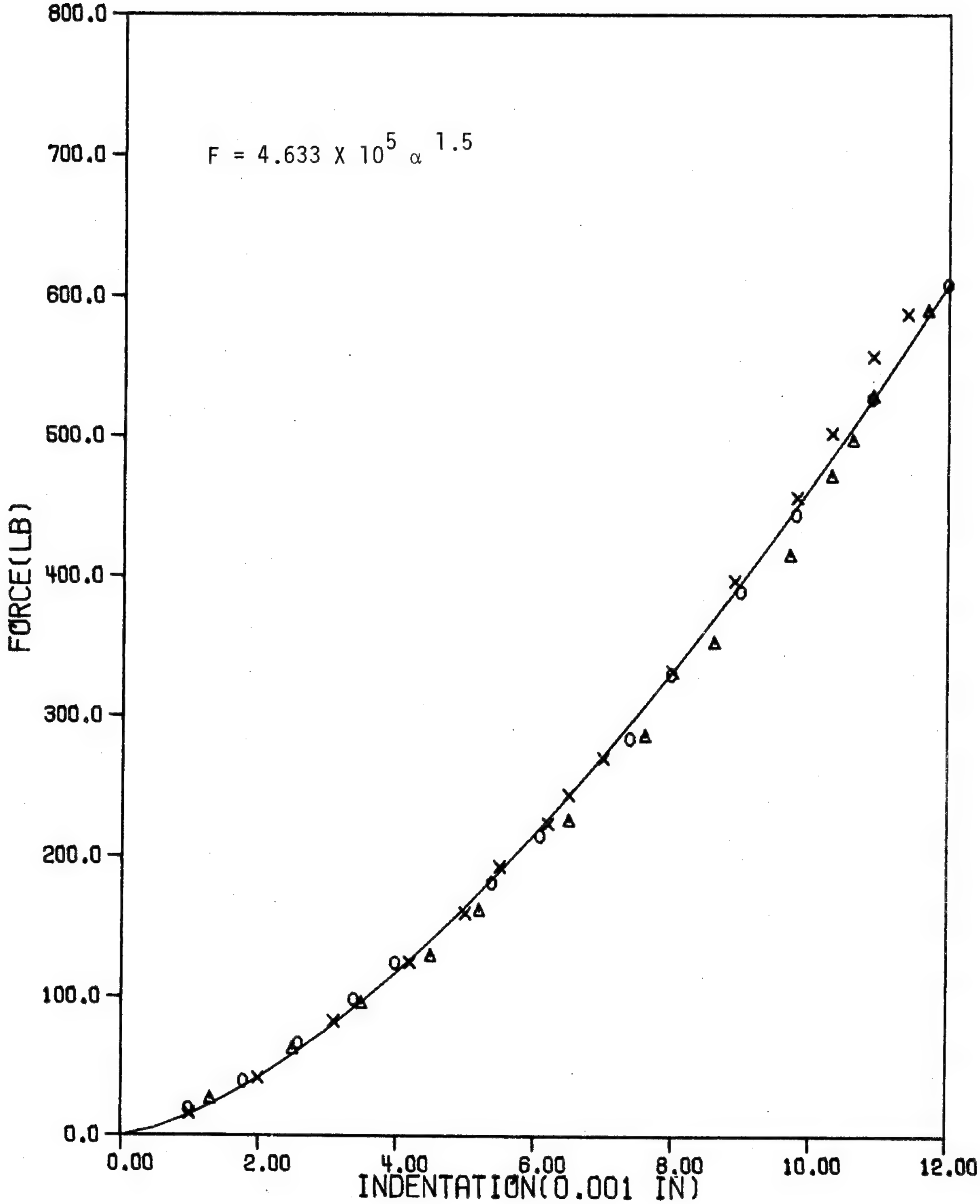

Fig. 2.6. Least-square fit with n = 1.5 for glass/epoxy with 4-inch span.

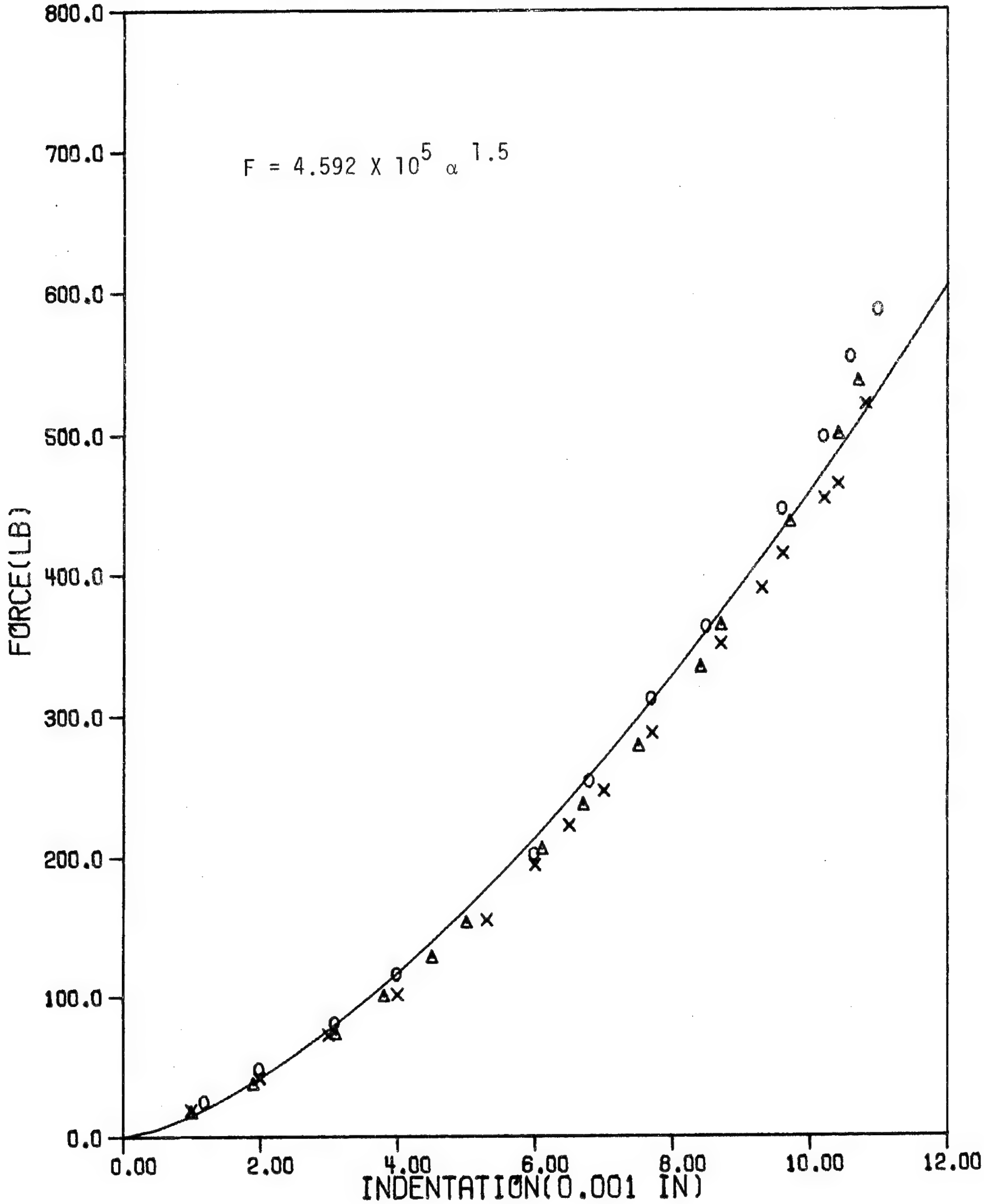

Fig. 2.7. Least-square fit with n = 1.5 for glass/epoxy with 6-inch span.

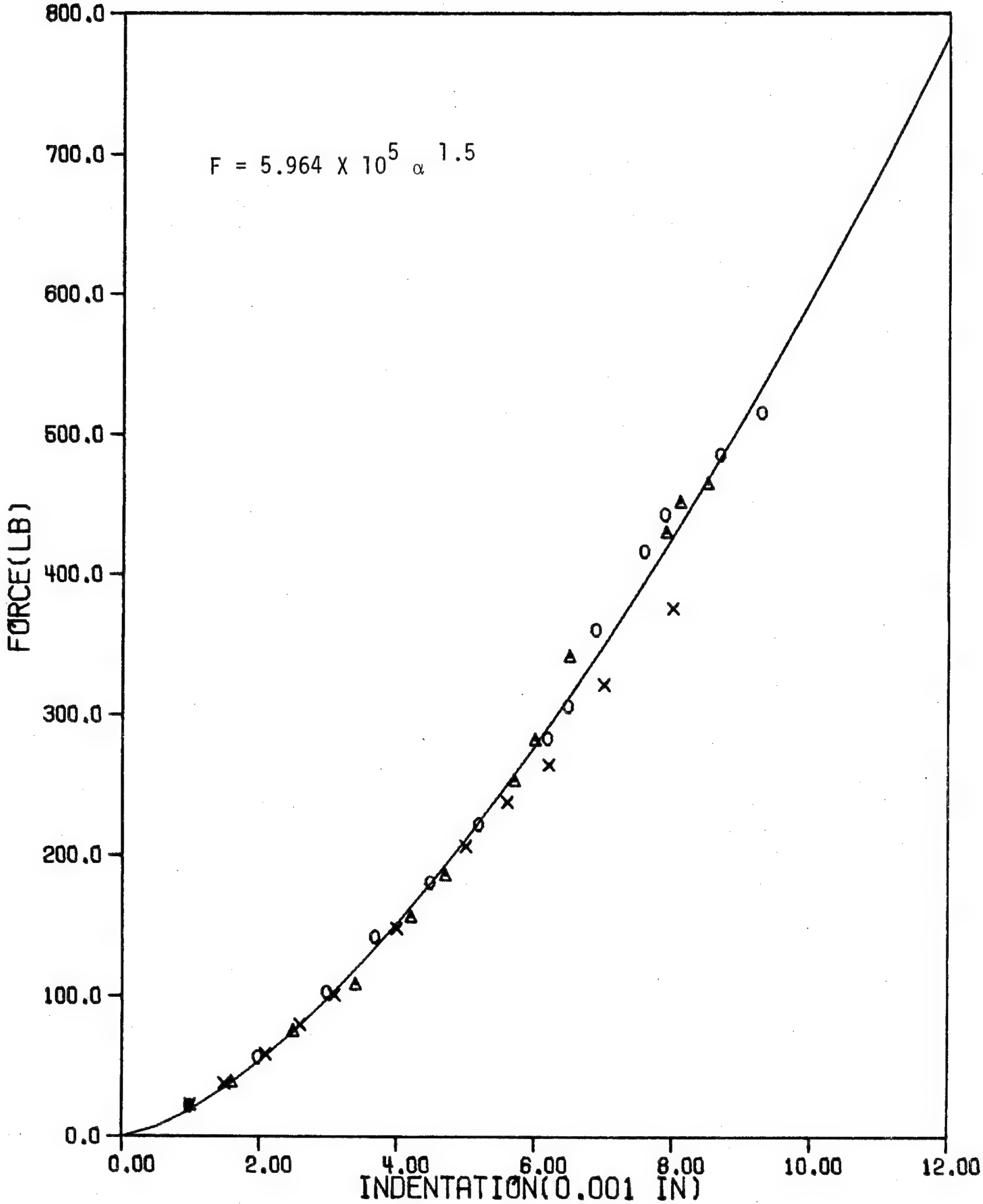

Fig. 2.8. Least-square fit of the contact force - indentation relation for graphite/epoxy with 2-inch span.

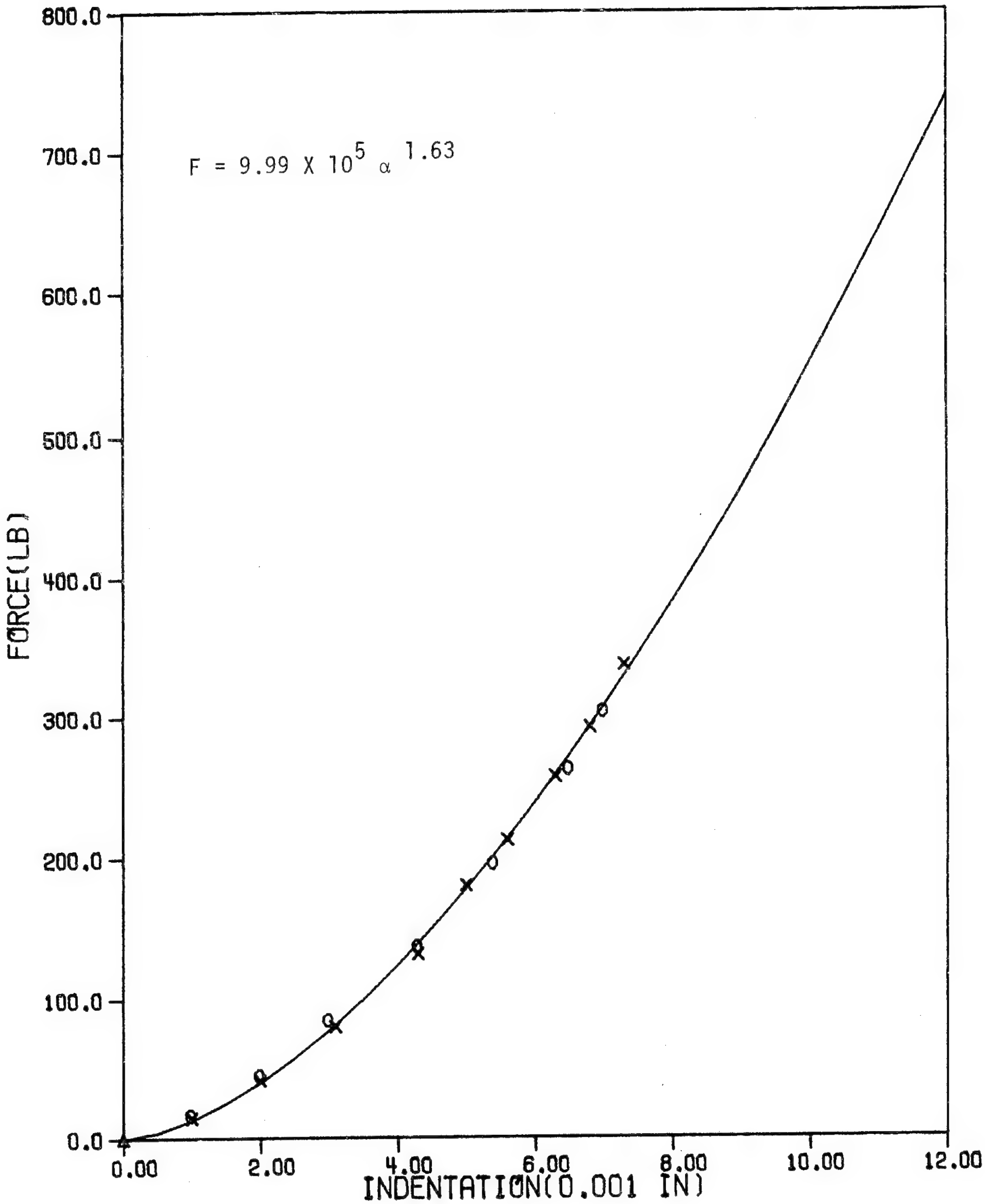

Fig. 2.9. Least-square fit of the contact force - indentation relation for graphite/epoxy with 4-inch span.

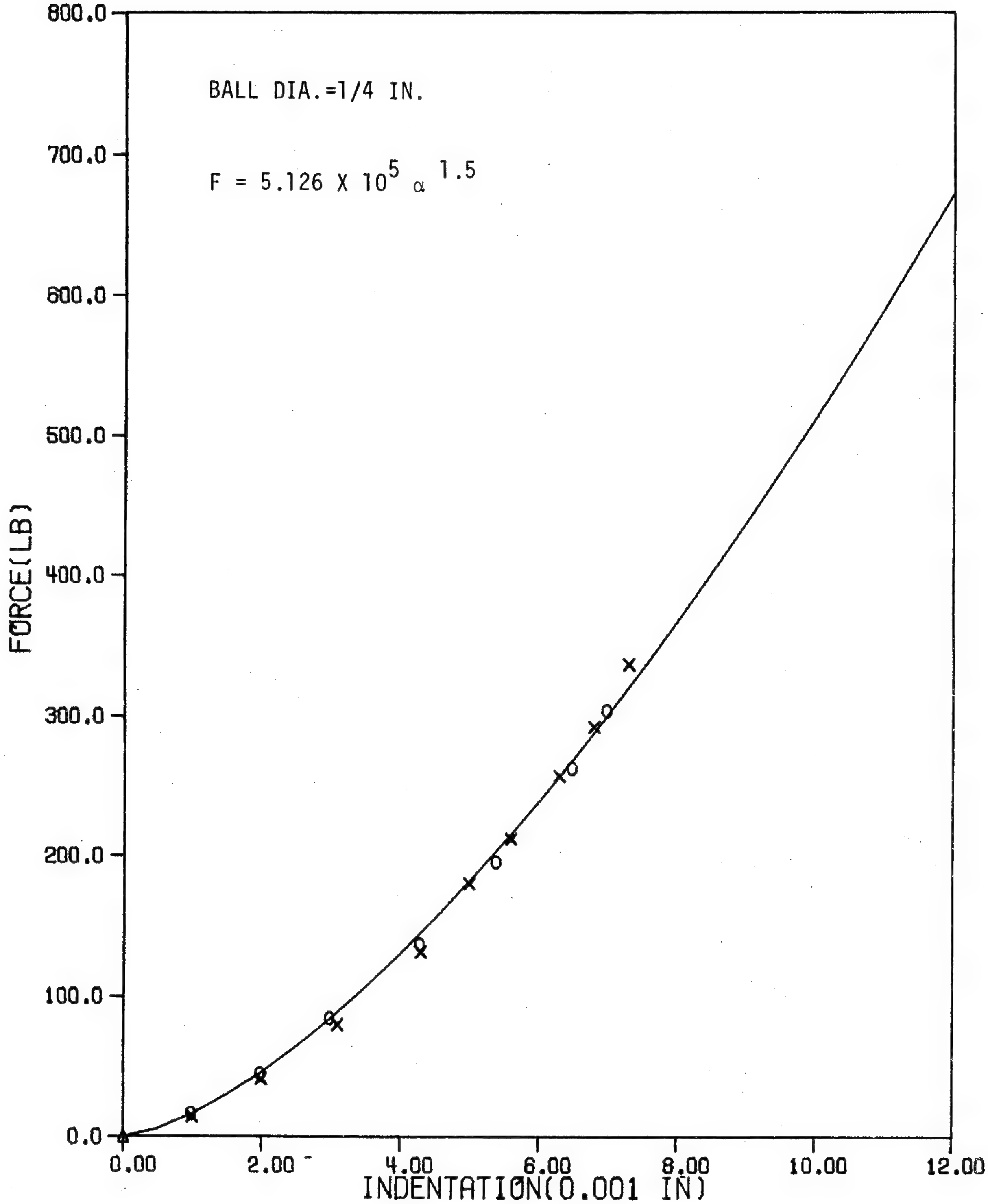

Fig. 2.10. Least-square fit with $n = 1.5$ for graphite/epoxy with 4-inch span.

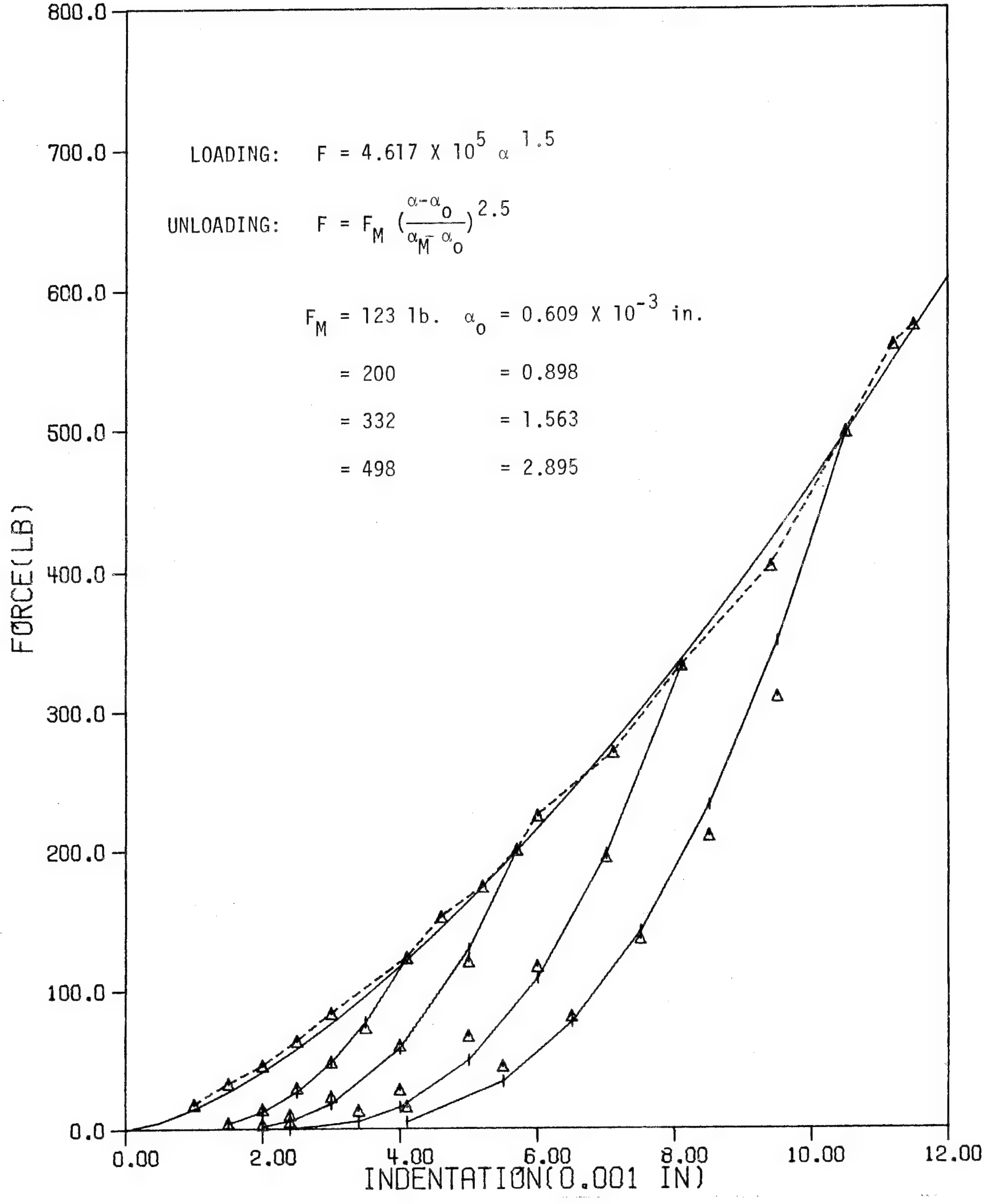

Fig. 2.11. Unloading curves for glass/epoxy with 2-inch span.

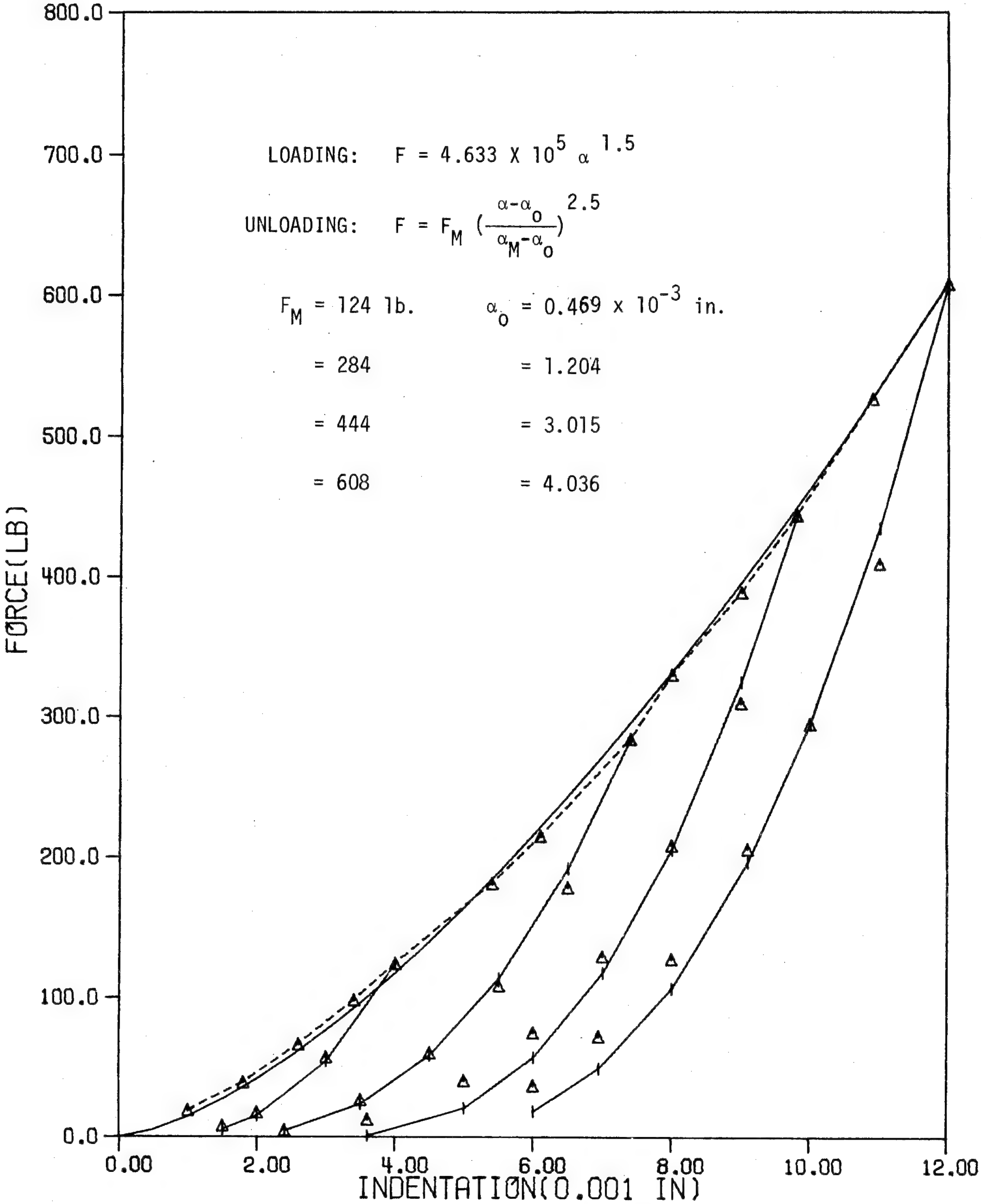

Fig. 2.12. Unloading curves for glass/epoxy with 4-inch span.

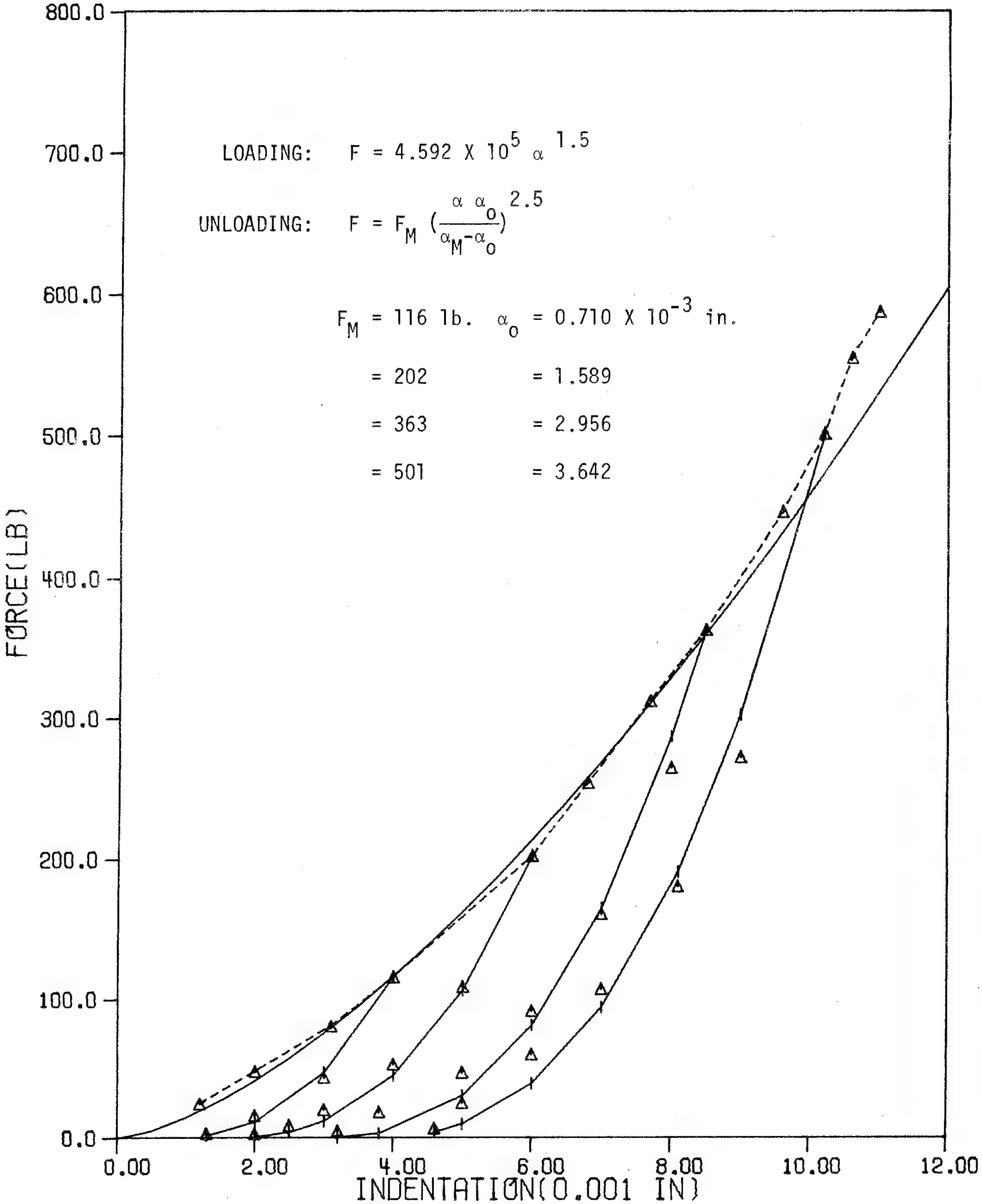

Fig. 2.13. Unloading curves for glass/epoxy with 6-inch span.

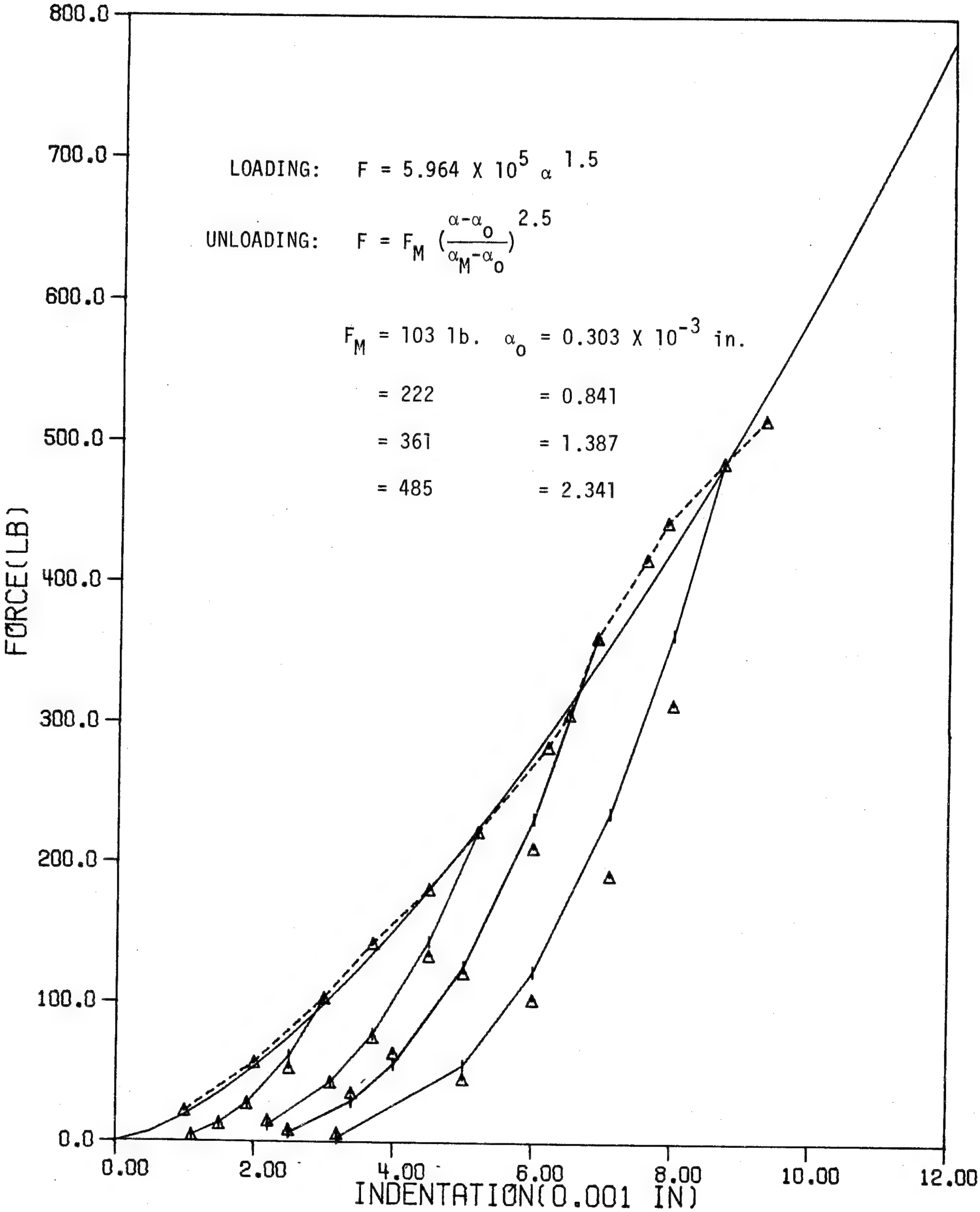

Fig. 2.14. Unloading curves for graphite/epoxy with 2-inch span.

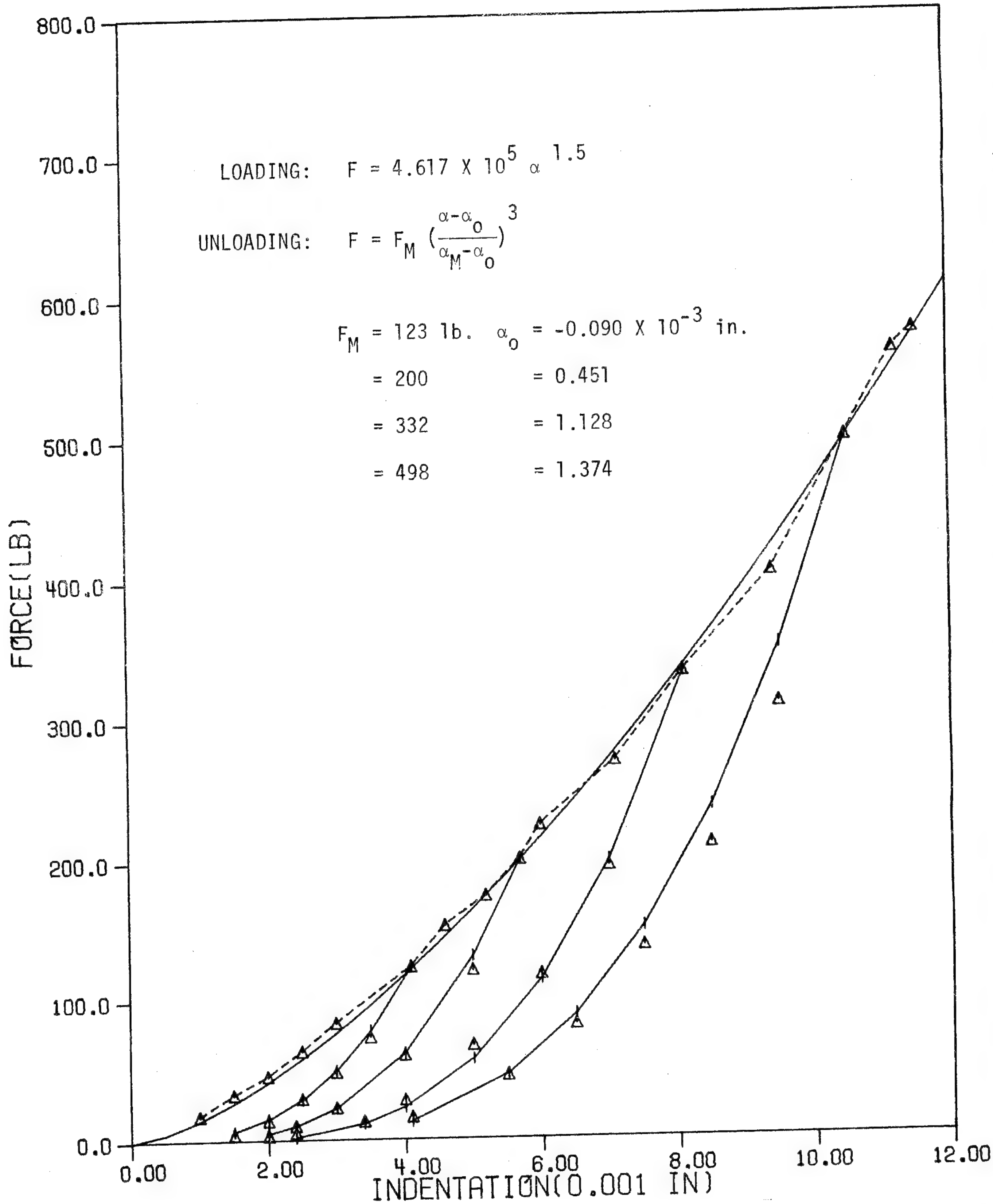

Fig. 2.15. Unloading curves for glass/epoxy with 2-inch span.

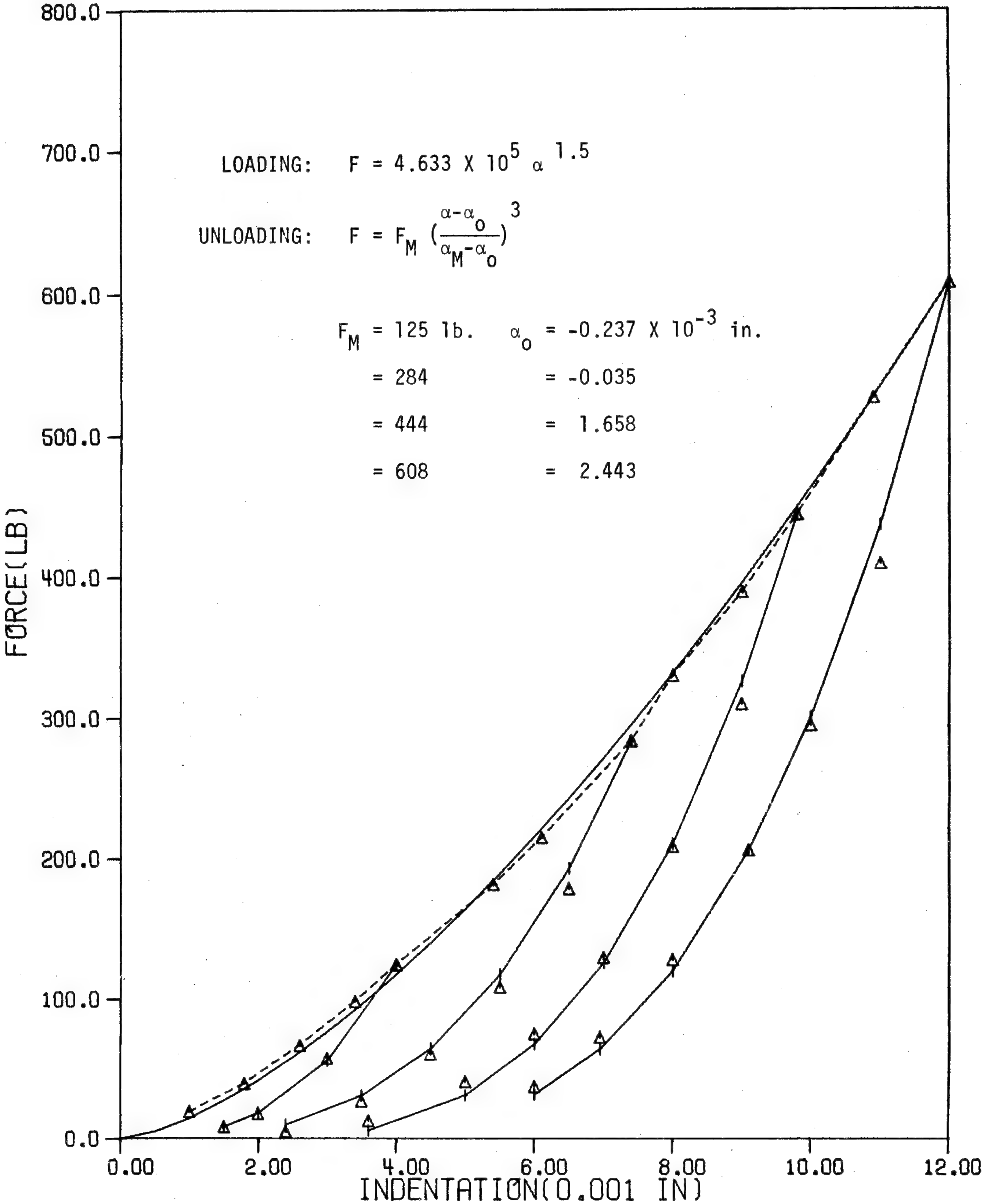

Fig. 2.16. Unloading curves for glass/epoxy with 4-inch span.

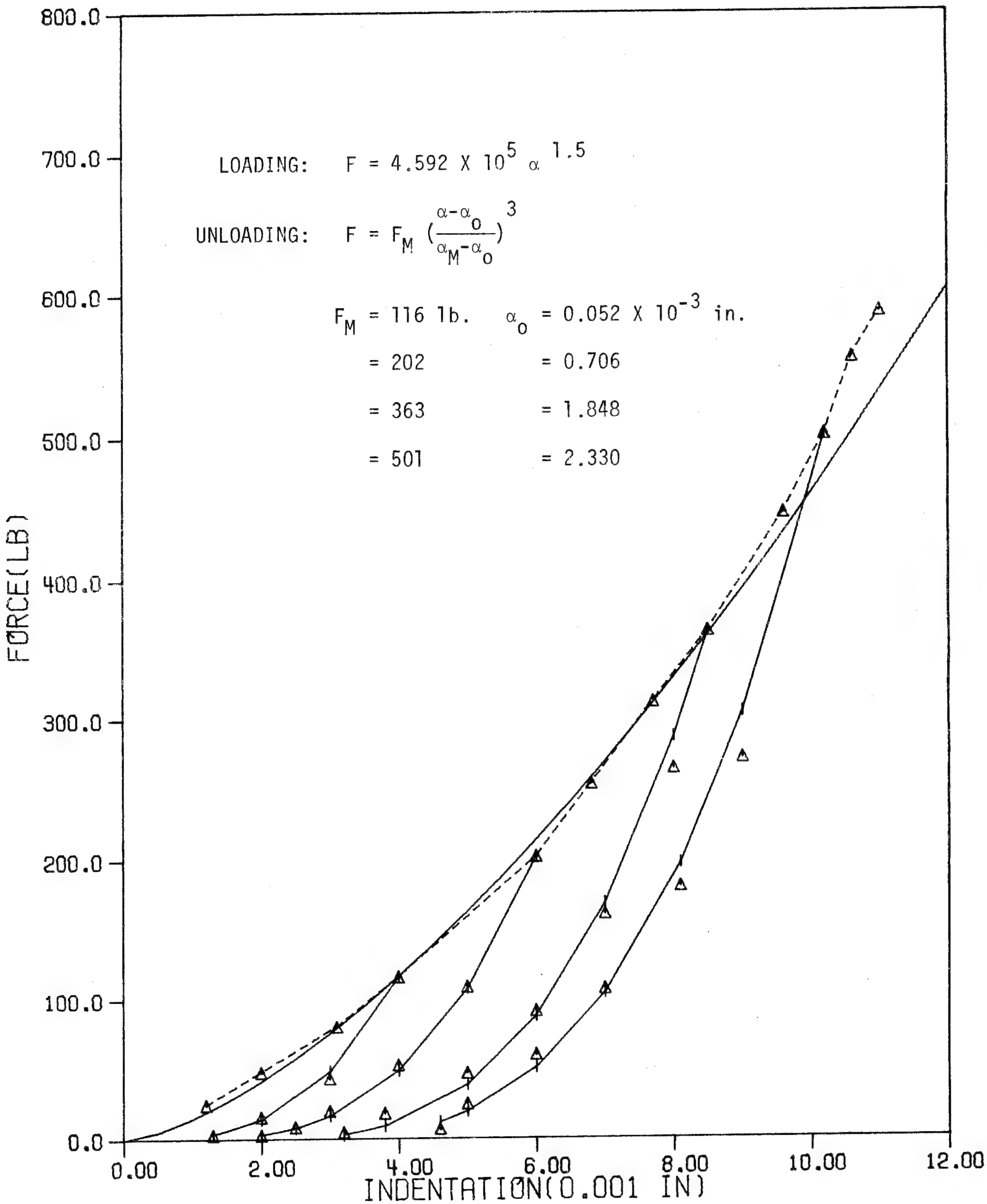

Fig. 2.17. Unloading curves for glass/epoxy with 6-inch span.

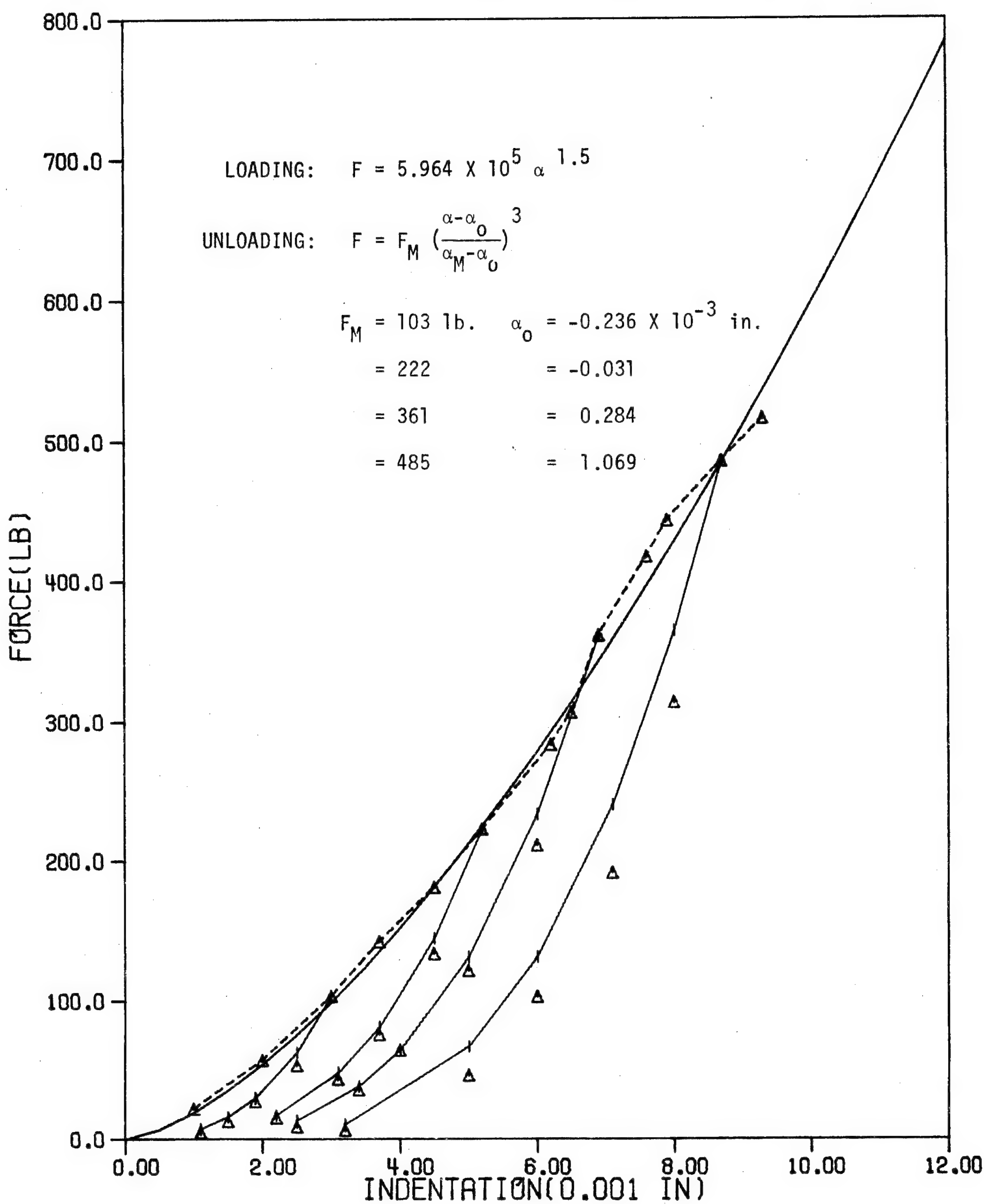

Fig. 2.18. Unloading curves for graphite/epoxy with 2-inch span.

3. IMPACT RESPONSES BY FINITE ELEMENT ANALYSIS

3.1 The Finite Element

A beam finite element with six degrees of freedom has been developed for the dynamic response of elastic isotropic beams subjected to impulsive loadings [5]. This high order beam element has been shown to be more efficient than the conventional element with four degrees of freedom. The element displacement function is taken as

$$v = a_1 + a_2x + a_3x^2 + a_4x^3 + a_5x^4 + a_6x^5 \tag{3-1}$$

where v is the transverse displacement and a_i are constant coefficients. The three degrees of freedom at each node are the transverse displacement v, the rotation θ, and the curvature κ. The coefficients a_i in Eq. (3-1) can be replaced by the six generalized nodal displacements at the two end nodes and, as a result, the displacement function can be alternatively expressed in terms of the nodal displacements.

The stiffness and mass matrices corresponding to the element displacement function has been presented elsewhere [5] and are reproduced in the following:

$$[k] = \frac{E_b I}{70L^3}\begin{bmatrix} 1200 & 600L & 30L^2 & -1200 & 600L & -30L^2 \\ & 384L^2 & 22L^3 & -600L & 216L^2 & -8L^3 \\ & & 6L^4 & -30L^2 & 8L^3 & L^4 \\ & & & 1200 & -600L & 30L^2 \\ & & & & 384L^2 & -22L^3 \\ & & & & & 6L^4 \end{bmatrix} \tag{3-2}$$

$$[m] = \frac{\rho A L}{55440} \begin{bmatrix} 21720 & 3732L & 281L^2 & 6000 & -1812L & 181L^2 \\ & 832L^2 & 69L^3 & 1812L & -532L^2 & 52L^3 \\ & & 6L^4 & 181L^2 & -52L^3 & 5L^4 \\ & & & 21720 & -3732L & 281L^2 \\ & & & & 832L^2 & -69L^3 \\ & & & & & 6L^4 \end{bmatrix} \quad (3\text{-}3)$$

where E_bI is the beam bending rigidity, L is the length, ρ is the mass density, and A is the cross-sectional area. If the finite element is to be used for the analysis of lamianted composite beams, then the bending rigidity E_bI has to be replaced by the equivalent bending rigidity D.

3.2 Impact Response

Based upon the stiffness and mass matrices given by Eqs. (3-2) and (3-3), respectively, a computer program has been written specifically for the dynamic response of a beam subjected to transverse impact of an elastic sphere. A finite difference scheme suggested by Wilson and Clough [6] was used to integrate the time variable in the equations of motion. In [5], the classical Hertzian law of elastic contact was used to solve a few example problems and excellent results were found.

The finite element program has been modified for the analysis of impact of laminated beams. The Hertzian indentation laws, Eqs. (2-1) with Eq. (2-2) or Eq. (2-3), as well as the measured indentation formulas can be chosen for the analysis. Both elastic loading and actual loading paths can be

incorporated in the program. The computer program with a brief user's instructions is presented in Appendix A.

Figures 3.1 - 3.4 show results for some example problems of simply-supported steel beams, subjected to impact of a steel ball. The diameter of the ball is $\frac{1}{2}$ in. The classical Hertzian law of contact was used in the computation. The material constants used are given by Eq. (4-31). From these results it can be seen that the impact velocity has a great effect on the maximum contact force and contact duration. The thickness of the beam has little effect for the two beam depths studied.

As reported in Section 2, a contact of the steel ball and the glass/epoxy and graphite/epoxy composite always results in a permanent deformation. The unloading paths are substantially different from the loading path. If the actual unloading paths are used, the contact force is certainly expected to deviate from that obtained by following elastic unloadings.

Figures 3.5 and 3.6 present the results for a glass/epoxy laminated composite beam with the dimension 0.19 in. D x 1.0 in. W x 7.5 in. L. This is the composite beam used for the indentation test. The actual indentation law with $k = 4.62 \times 10^5$ and $n = 1.5$ for loading and $q = 2.5$ for unloading was used for the computation. Note that, in this case, the steel ball has a diameter of 1/4 in. same as that of the identor in the static indentation test. The material constants for composite are

$$E_L = 5.7 \times 10^6 \text{ psi}$$

$$E_T = 1.2 \times 10^6 \text{ psi}$$

$$G_{LT} = 0.6 \times 10^6 \text{ psi}$$

$$\nu_{LT} = 0.26$$

$$\rho = 0.002016 \text{ slug/in}^3 \quad (0.000168 \text{ lb-sec}^2/\text{in}^4)$$

From the results in these figures it can be seen that the contact force drops more rapidly after reaching its maximum value when the inelastic unloading path is followed. However, the total contact duration does not seem to be affected by the inelastic unloading.

The finite element program developed here can also be used in conjunction with the experimentally obtained contact law to compute the dynamic strain at any point on the beam. The dynamic strain can be experimentally measured by using a strain gage. By comparing the measured strain and that predicted by the finite element solution, it may be possible for us to determine the effect of a result of this comparison. The static indentation law may be modified to account for the strain rate effect. The result of the comparison will be reported in the future.

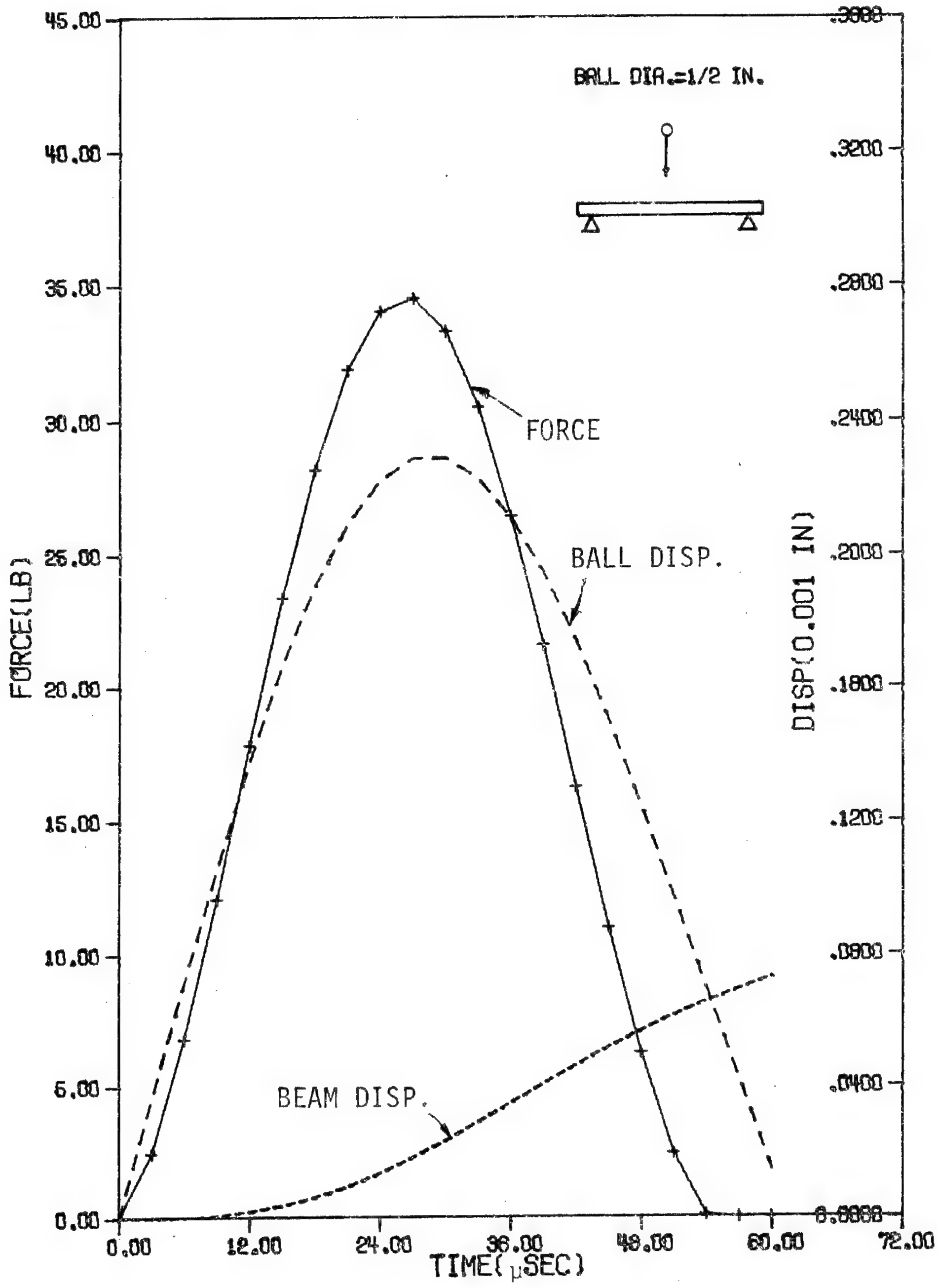

Fig. 3.1. Response of simply-supported steel beam (0.5"W X 0.5"D X 30"L) subjected to impact of a steel ball with initial velocity 12 in/sec.

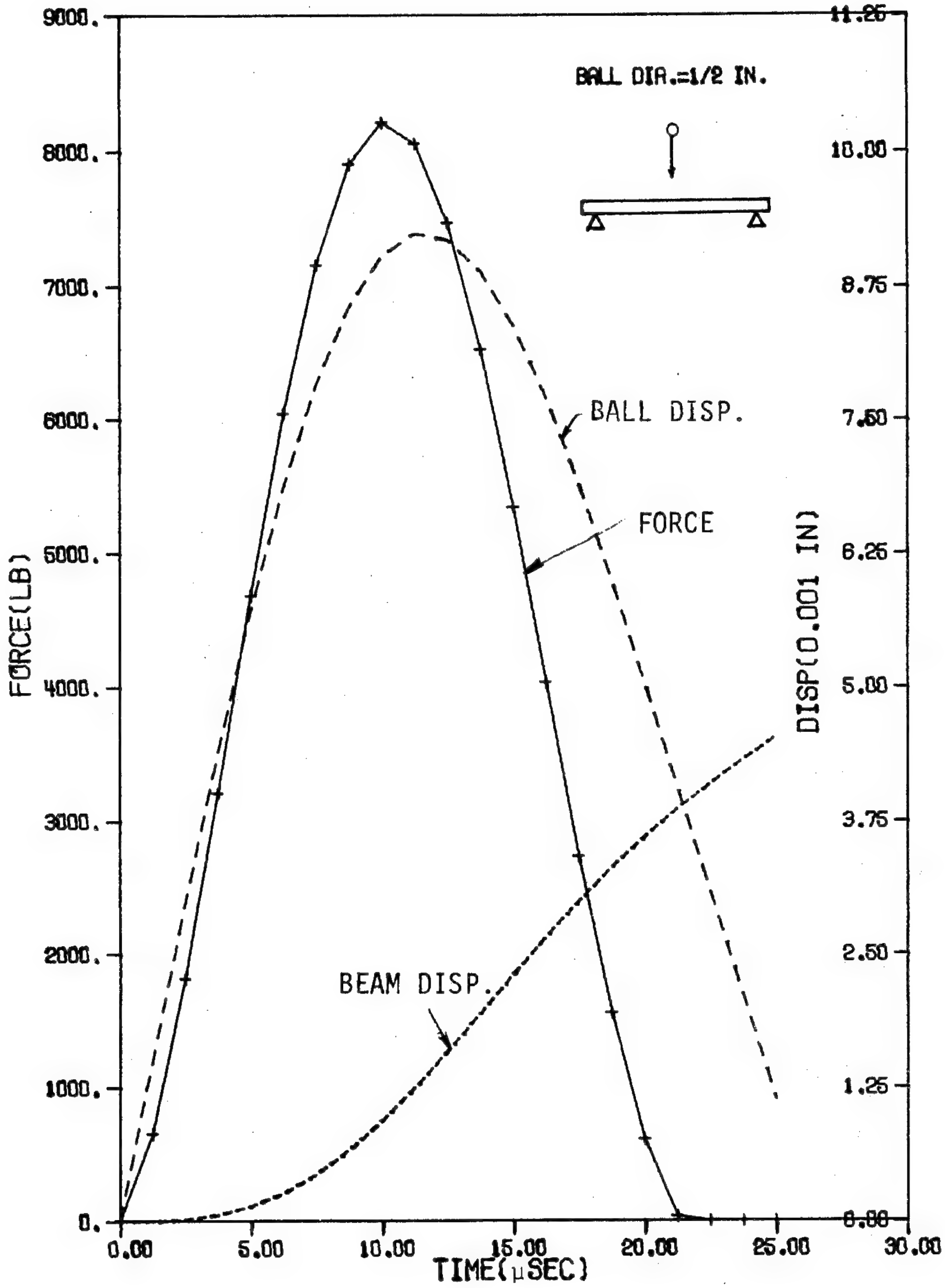

Fig. 3.2. Response of a simply-supported steel beam (0.5"W X 0.5"D X 30"L) subjected to impact of a steel ball with initial velocity 1200 in/sec.

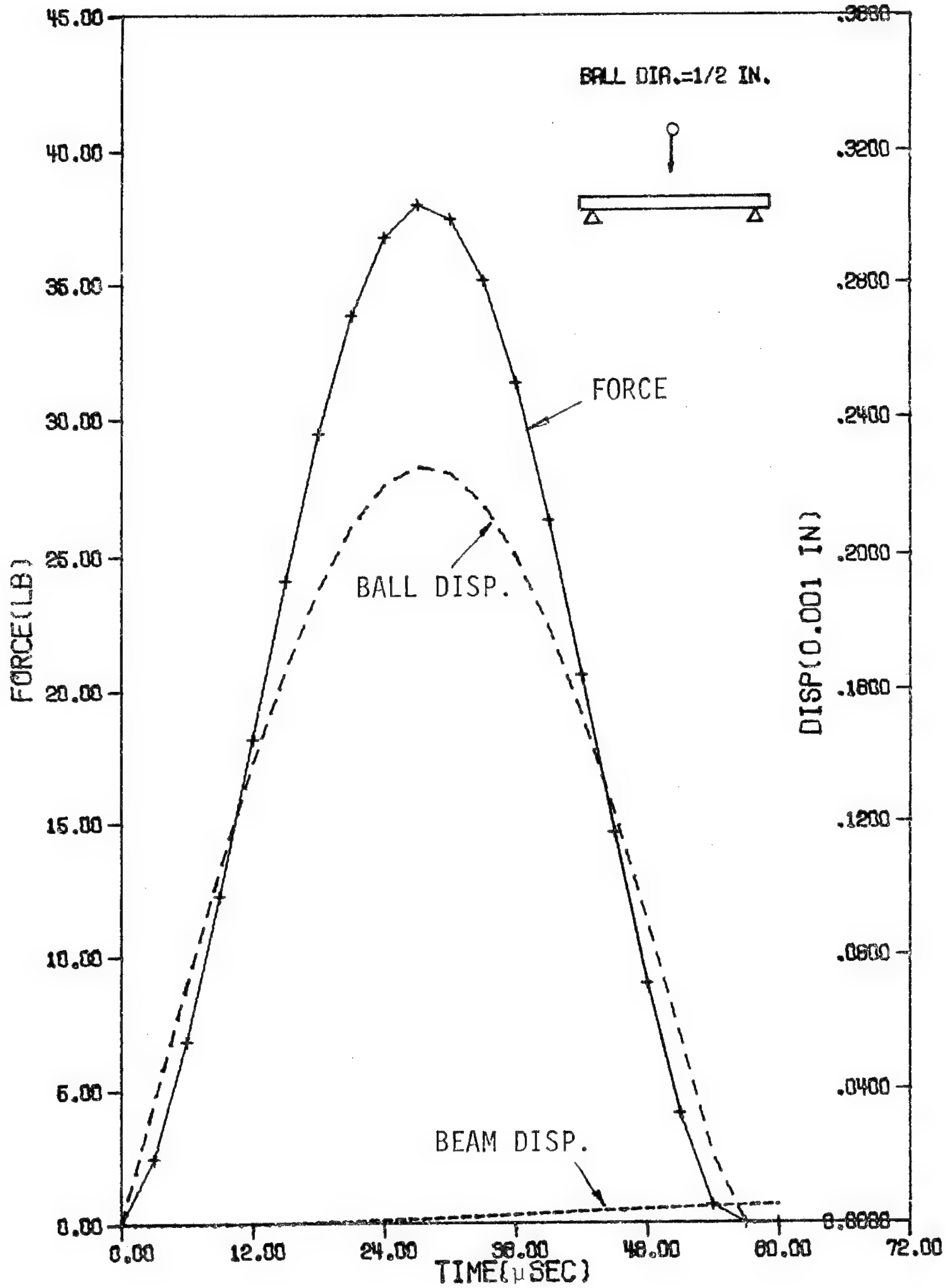

Fig. 3.3. Response of a simply-supported steel beam (0.5"W X 3.0"D X 30"L) subjected to impact of a steel ball with initial velocity 12 in/sec.

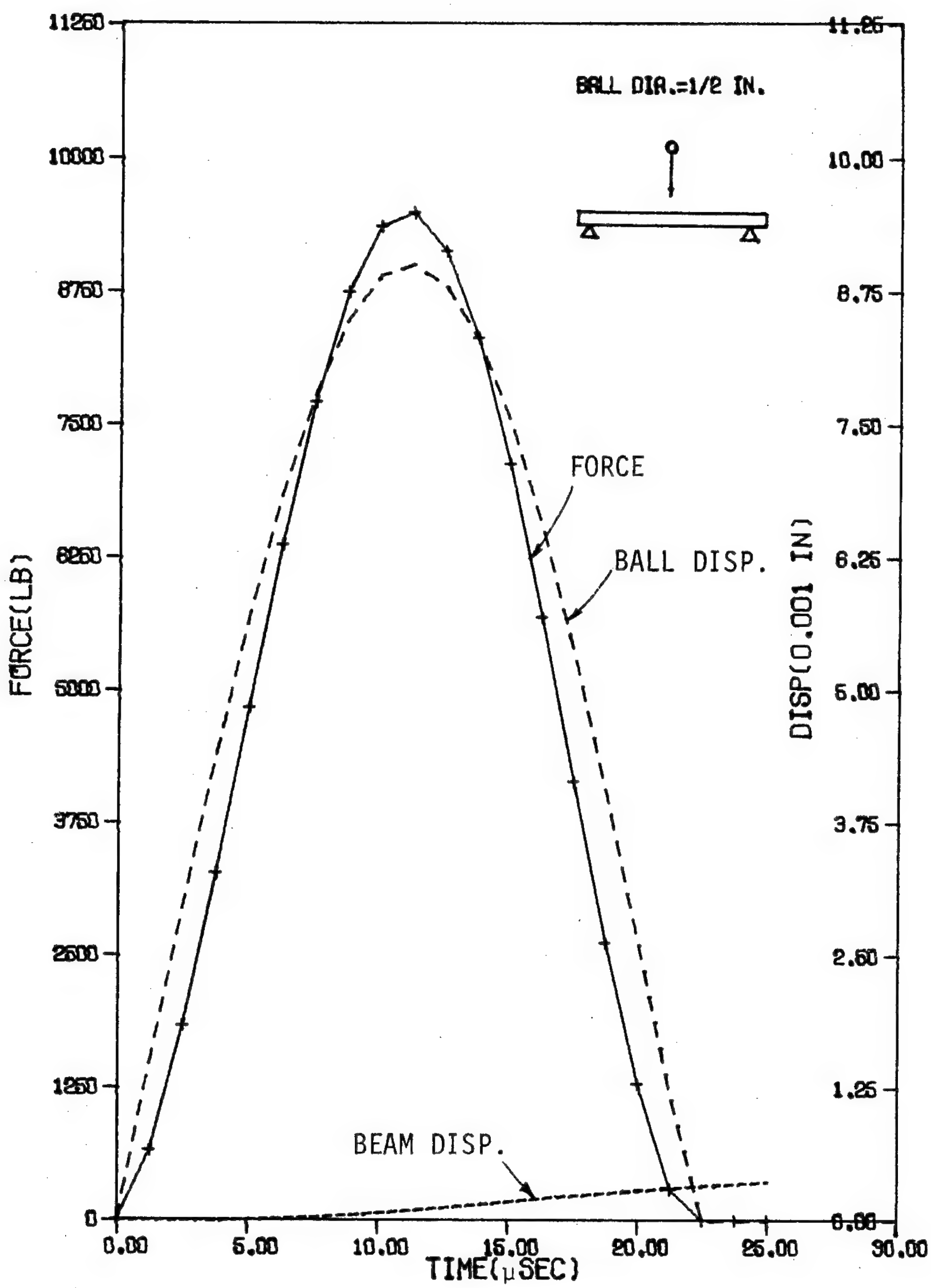

Fig. 3.4. Response of a simply-supported steel beam (0.5"W X 3"D X 30"L) subjected to impact of a steel ball with initial velocity 1200 in/sec.

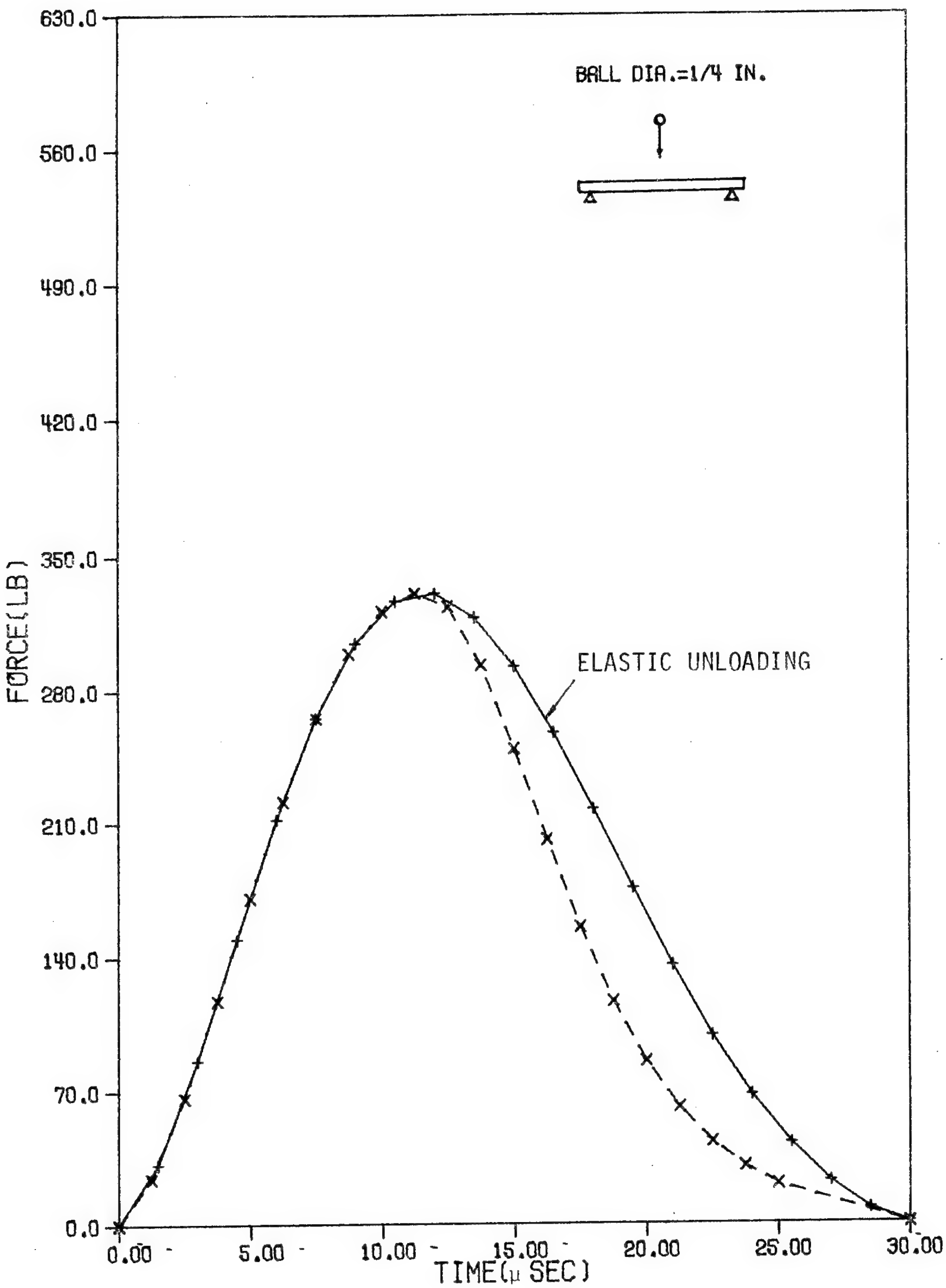

Fig. 3.5. Contact forces with elastic and inelastic unloadings in a simply-supported glass/epoxy laminated beam (1"W X 0.19"D X 7.5"L) subjected to impact of a steel ball at v_i = 1000 in/sec.

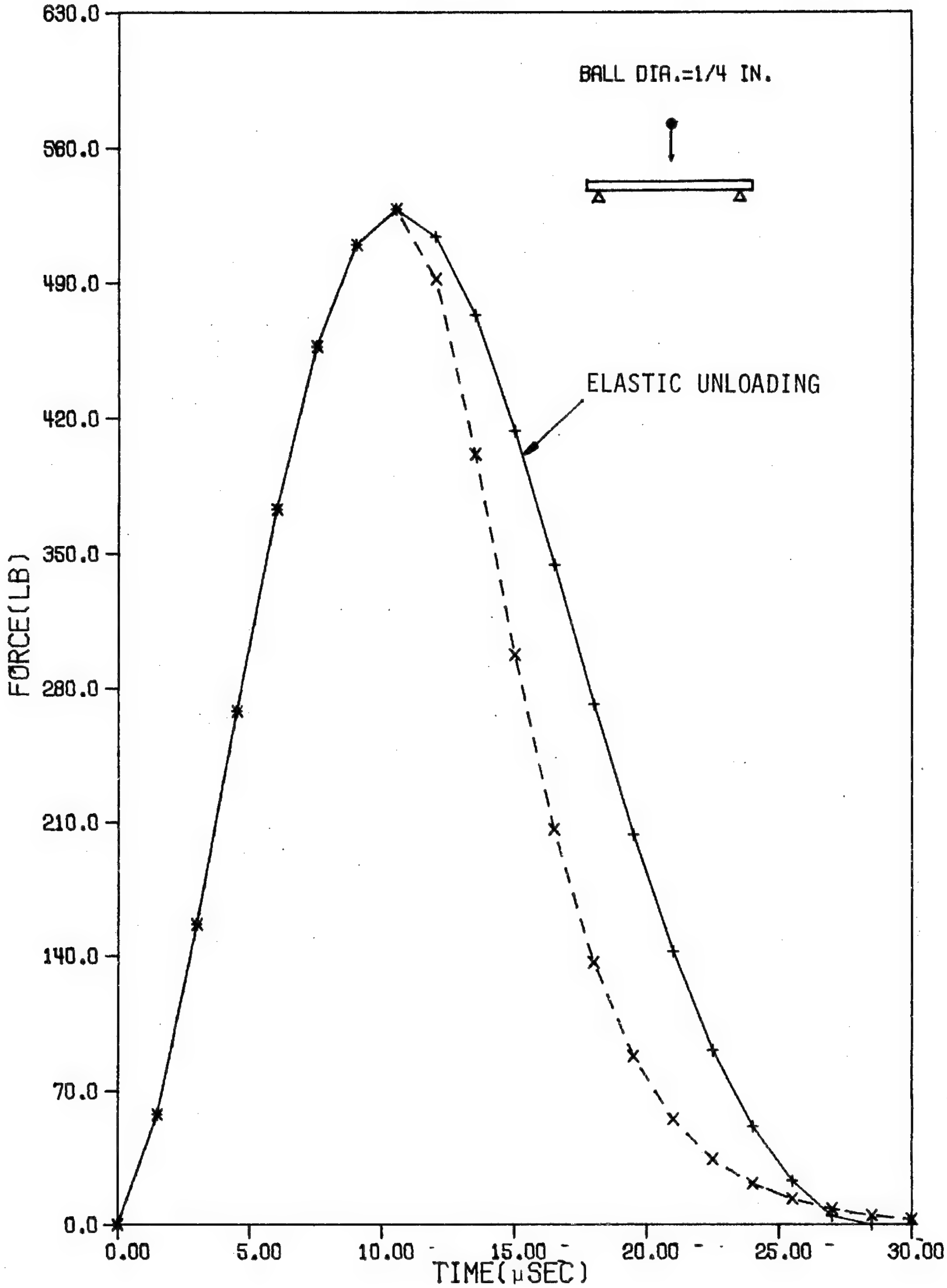

Fig. 3.6. Contact forces with elastic and inelastic unloadings in a simply-supported glass/epoxy laminated beam (1"W X 0.19"D X 7.5"L) subjected to impact of a steel ball at v_i = 1500 in/sec.

4. A Simple Method for Computing Contact Force and Duration in Elastic Impact

In using the finite element program described in Section 3, we have to choose a proper time increment Δt and the total length of time integration prior to the solution. A poor choice of Δt may result in poor finite difference solutions. A simple way to obtain an approximate impact duration prior to the use of the finite element program certainly will avoid futile trials. In the following, a simple method is developed for computing an approximate contact force and the contact duration.

4.1 Impact of an Elastic Sphere on a Mass with a Flat Surface

A simple analysis for a spherical projectile impacting an elastic mass with a flat surface was proposed by Timoshenko [7] as follows. Denoting the mass and velocity of the target by m_t and v_t, respectively, and the mass and the velocity of the sphere by m_s and v_s, respectively, the rates of change of velocity during impact are

$$m_t \frac{dv_t}{dt} = F \tag{4-1}$$

$$m_s \frac{dv_s}{dt} = - F \tag{4-2}$$

where F is the contact force. The velocity of the relative approach α (the indentation) is

$$\dot{\alpha} = v_s - v_t \tag{4-3}$$

From Eqs. (4-1) to (4-3), we obtain

$$\ddot{\alpha} = - F \frac{m_t + m_s}{m_t m_s} \tag{4-4}$$

Substituting the Hertz law of contact, Eq. (2-1), in Eq. (4-4), we obtain

$$\ddot{\alpha} = - k\xi\alpha^{3/2} \tag{4-5}$$

where

$$\xi = \frac{1}{m_t} + \frac{1}{m_s} \tag{4-6}$$

Integrating Eq. (4-5), we have

$$\frac{1}{2}(\dot{\alpha}^2 - v_i^2) = - \frac{2}{5} k\xi\alpha^{5/2} \tag{4-7}$$

The maximum value of α, α_{max}, occurs at $\dot{\alpha} = o$. We obtain

$$\alpha_{max} = \left(\frac{5}{4} \; \frac{v_i^2}{k\xi} \right)^{2/5} \tag{4-8}$$

This together with the Hertzian law yields the maximum contact force.

From Eq. (4-7), the following relation is derived:

$$dt = \frac{d\alpha}{(v_i^2 - \frac{4}{5} k\xi\alpha^{5/2})^{1/2}} \tag{4-9}$$

By introducing

$$\eta = \left(\frac{4}{5}\frac{k\ \xi}{v_i^2}\right)^{2/5} \alpha = \frac{\alpha}{\alpha_{max}} \tag{4-10}$$

Equation (4-9) can be rewritten as

$$dt = \frac{\alpha_{max}}{v_i} \; \frac{d\eta}{(1 - \eta^{5/2})^{1/2}} \tag{4-11}$$

If we assume that the maximum indentation, α_{max}, is achieved half way through the entire contact, then the duration of impact is obtained from

integrating Eq. (4-11) as

$$T = \frac{2\alpha_{max}}{v_i} \int_0^1 \frac{d\eta}{(1 - \eta^{5/2})^{1/2}} = 2.94 \frac{\alpha_{max}}{v_i} \tag{4-12}$$

4.2 Equivalent Mass Model

In view of the simple formulas given by Eqs. (4-8) and (4-12), we will attempt to find an equivalent mass m_t to represent an actual beam or plate. Once this is accomplished, the maximum contact force and the contact duration can be estimated easily.

The equivalent system is developed based upon the condition that it stores the same amount of kinetic and strain energies as in the actual system. It is assumed that in both systems the strain energies in the impactors are negligible and that the kinetic energies are identical. It is also assumed that the spheres do the same amount of work on both the actual and the equivalent targets. With these assumptions, we conclude that the total kinetic energy of the equivalent mass, K_t, should be equal to the kinetic energy K plus the strain energy U of the actual elastic target, i.e.,

$$K + U = K_t \tag{4-13}$$

The kinetic energy in the equivalent target system is simply

$$K_t = \frac{1}{2} m_t v_t^2 \tag{4-14}$$

From Eq. (4-1), the velocity of the equivalent mass can be obtained by integration as

$$v_t = -\frac{1}{m_t} \int_o^t F(\tau)\, d\tau \tag{4-15}$$

From all the previous studies, the contact force history resembles a sine function. In view of this, we approximate the contact force as follows

$$F = F_{max} \sin(\pi t/T) \tag{4-16}$$

Substituting Eq. (3-16) into Eq. (3-15) and integrating from t=o to t = T/2 we obtain the velocity of the equivalent target at t = T/2 as

$$v_t = -\frac{1}{m_t} \frac{T}{\pi} F_{max} \tag{4-17}$$

Substitution of Eq. (4-17) into Eq. (4-14) and then into Eq. (4-13) lead to

$$(K + U)_{t=T/2} = \frac{1}{2} \frac{1}{m_t} (\frac{T}{\pi})^2 F_{max}^2 \tag{4-18}$$

Since the deflection of the beam is proportional to the applied force F, both U and K contain F_{max}^2 terms and can be factored out as

$$U = F_{max}^2 U^* , \quad K = F_{max}^2 K^* \tag{4-19}$$

in which U* and K* do not depend on F_{max}. Equation (3-18) can now be written as

$$\frac{T^2}{2m_t\pi^2} = (U^* + K^*)_{t=T/2} \tag{4-20}$$

From Eqs. (4-6), and (4-8) and (4-12), we note that the contact duration T is a function of the equivalent mass m_t. Thus, Eq. (4-20) is basically a nonlinear equation for m_t. Numerical methods will be used to find solutions for this equation.

4.3 Simply-Supported Beam

Consider a beam of cross-sectional area A and bending rigidity D. The equation of motion is

$$D \frac{\partial^4 w}{\partial x^4} + \rho A \frac{\partial^2 w}{\partial t^2} = q(x,t) \tag{4-21}$$

where ρ is the average mass density (over the thickness) and $q(x,t)$ is a time dependent forcing function. For a homogeneous elastic beam, we have

$$D = EI \tag{4-22}$$

For laminated composite beams, D is estimated according to Eq. (4-36).

If the force is a concentrated force $F(t)$ applied at $x=c$, then the solution for Eq. (4-21) can be expressed as [8]

$$w(x_1 t) = \frac{1}{\rho A} \sum_{n=1}^{\infty} \frac{w_n(x) w_n(c)}{\omega_n \int_0^L w_n^2(x) dx} \int_0^t F(\tau) \sin \omega_n (t-\tau) d\tau \tag{4-23}$$

where $w_n(x)$ is the shape function for the nth natural mode of vibration, and ω_n is the corresponding natural frequency.

For a simply-supported beam, we obtain

$$w_n(x) = \sin \frac{n\pi x}{L} \tag{4-24}$$

and

$$\omega_n^2 = \left(\frac{n\pi}{L}\right)^4 \frac{D}{\rho A} \tag{4-25}$$

If the concentrated force is given by Eq. (4-16), then the beam deflection w can be obtained from Eq. (4-23) as

$$w(x,t) = \frac{2F_{max} L^3}{\pi^4 D} \sum_{n=1}^{\infty} w_n(c) \frac{1}{n^4} \left[\frac{4T^2}{4T^2 - T_n^2} \left(\sin \frac{\pi}{T} t - \frac{T_n}{2T} \sin \omega_n t\right) \right] w_n(x) \ , \text{ for } t \leq T \tag{4-26}$$

In Eq. (4-26),

$$T_n = \frac{2\pi}{\omega_n} \tag{4-27}$$

is the period for the nth mode. The strain energy and the kinetic energy can be computed in a straightforward manner. We obtain at $t = T/2$

$$U^* = \frac{16L^3T^4}{\pi^4 D} f_1$$

$$K^* = \frac{16\rho AL^7T^2}{\pi^6 D^2} g_1 \tag{4-28}$$

where

$$f_1 = \sum_{n=1}^{\infty} \left\{ \frac{n^2}{4n^4T^2 - T_1^2} \left[1 - \frac{T_1}{2n^2 T} \sin(n^2 \frac{\omega_1 T}{2}) \right] w_n(c) \right\}^2 \tag{4-29}$$

$$g_1 = \sum_{n=1}^{\infty} \left\{ \frac{1}{4n^4T^2 - T_1^2} \cos(n^2 \frac{\omega_1 T}{2}) w_n(c) \right\}^2 \tag{4-30}$$

From the numerical examples, it has been observed that use of fifty terms in the series in Eqs. (4-29) and (4-30) should provide a converged

solution. In all examples presented in this section, the classical Hertzian law is used.

As a first evaluation of the equivalent mass concept, we consider a problem solved by Timoshenko [9] using a numerical procedure to solve a nonlinear integral equation. The steel beam considered has a 0.39 in. x 0.39 in. (1 cm x 1 cm) cross-section and 6.04 in. (15.35 cm) length. The beam is simply-supported at two ends and subjected to impact of a steel ball with 0.79 in. (2 cm) diameter. The material properties are

$$\begin{aligned} E &= 31 \times 10^6 \text{ psi} \\ \nu &= 0.29 \\ \rho &= 0.00894 \text{ slug/in}^3 \ (0.000745 \text{ lb-sec}^2/\text{in}^4) \end{aligned} \qquad (4\text{-}31)$$

It should be pointed out that in the numerical computation, the value for the mass density as given by Eq. (4-31) should be divided by a factor of 12 if the length is given in inches.

Figure 4.1 shows the contact force histories according to Timoshenko's solution and the equivalent mass model. Excellent agreement is noted.

Figures 4.2 and 4.3 show the contact forces of a simply-supported steel beam subjected to impact of a steel ball of 1/2 in. diameter with different velocities. The beam has a 1/2 in. x 1/2 in. cross-section and is 30 in. long. Both the equivalent mass model results and the finite element results are found to have a very close agreement.

The results for a thicker steel beam (1/2 in. W x 3 in. D x 30 in. L) with simple supports are presented in Figs. 4.4 and 4.5 for v_i = 12 in/sec. and 1200 in./sec., respectively. Again, the equivalent mass model works quite well in predicting the magnitude and duration of the contact force.

Figure 4.6, shows the results for a simple-supported thin steel beam (0.5 in. W x 0.08 in. D x 15 in. L) subjected to the impact of a steel ball of 0.5 in. diameter with v_i = 100 in./sec. The equivalent mass model is able to predict the maximum contact force but not the contact duration due to the long tail portion.

Figure 4.7 shows the contact force history for a composite beam of the same dimension and impact condition as the previous problem. The laminated beam consists of 16 piles of graphite/epoxy composite. The ply-thickness is 0.005 in. and the lay-up is $(0/90/0/90)_{2s}$. The material constants are

$$\begin{aligned} E_L &= 30 \times 10^6 \text{ psi}, \; E_T = 0.75 \times 10^6 \text{ psi} \\ G_{LT} &= 0.4 \times 10^6 \text{ psi}, \; \nu_{LT} = 0.25, \\ \rho &= 0.00178 \text{ slug/in}^3 \;\; (0.000148 \text{ lb-sec}^2/\text{in}^4) \end{aligned} \tag{4-32}$$

The modified Hertzian law of contact given by Eq. (2-3) was used for the solution. Again, from Fig. 4.7 we find that the eqivalent mass model is excellent in predicting the maximum contact force but poor in estimating the total contact time. From the numerical examples carried out, it seems that the equivalent mass model can not yield accurate contact time if the target is too thin.

4.4 Simply-Supported Rectangular Plate

The plate theory developed by Whitney and Pagano [10] for laminated composites is used for the analysis. This plate theory takes the transverse shear deformation into account and has been shown by Sun and Lai [11] to be adequate for wave propagation. For simplicity, only cross-ply laminated plates are considered, for which the equations of motion are given by

$$D_{11}\psi_{x,xx} + D_{66}\psi_{x,yy} + (D_{12} + D_{66})\psi_{y,xy} - \kappa A_{55}\psi_x - \kappa A_{55}w,_x = \rho I\ddot{\psi}_x \qquad (4\text{-}33)$$

$$(D_{12} + D_{66})\psi_{x,xy} + D_{66}\psi_{y,xx} + D_{22}\psi_{y,yy} - \kappa A_{44}\psi_y - \kappa A_{44}w,_y = \rho I\ddot{\psi}_y \qquad (4\text{-}34)$$

$$\kappa A_{55}\psi_{x,x} + \kappa A_{55}w,_{xx} + \kappa A_{44}\psi_{y,y} + \kappa A_{44}w,_{yy} + q = \rho h\ddot{w} \qquad (4\text{-}35)$$

where a comma indicates partial differentiation, q is the lateral load, w is the transverse displacement, ψ_x and ψ_y are the rotations of the plane sections, $\kappa(=\pi^2/12)$ is a shear correction factor, ρ is the average mass density (over the thickness), h is the plate thickness, I is the rotary inertia, and

$$(A_{ij}, D_{ij}) = \int_{-h/2}^{h/2} \bar{Q}_{ij}(1,z^2)dz \qquad (4\text{-}36)$$

In Eq. (4-36), $\bar{Q}_{ij}$ are the reduced stiffnesses for the composite material. For an isotropic elastic plate, the following relations exist:

$$\begin{aligned} D_{11} &= D_{22} = \frac{E\,h^3}{12(1-\nu^2)} \\ D_{12} &= \nu D_{11} \\ D_{66} &= \frac{1-\nu}{2} D_{11} \\ A_{11} &= A_{22} = \frac{Eh}{1-\nu^2} \\ A_{12} &= \nu A_{11} \\ A_{44} &= A_{55} = \frac{Eh}{2(1+\nu)} \end{aligned} \qquad (4\text{-}37)$$

The equations of motion given by Eqs. (4-33) to (4-35) reduce to those for the Mindlin's plate theory [12].

If we separate the total displacement into the bending part, w_b, and that due to the transverse shear deformation, w_s, then we have

$$\begin{aligned} \psi_x &= - w_{b,x} \\ \psi_y &= - w_{b,y} \qquad\qquad (4\text{-}38) \\ w &= w_b + w_s \end{aligned}$$

In terms of w_b and w_s, the equation of motion can be written as

$$D_{11} w_{b,xxx} + (D_{12} + 2D_{66})w_{b,xyy} + \kappa A_{55}w_{s,x} = \rho I\ddot{w}_{b,x} \qquad (4\text{-}39)$$

$$(D_{12} + 2D_{66})w_{b,xxy} + D_{22}w_{b,yyy} + \kappa A_{44}w_{s,y} = \rho I\ddot{w}_{b,y} \qquad (4\text{-}40)$$

$$\kappa A_{55}w_{s,xx} + \kappa A_{44}w_{s,yy} + q = \rho h(\ddot{w}_b + \ddot{w}_s) \qquad (4\text{-}41)$$

Combining equations (4-39) with (4-40), we have

$$\begin{aligned} &D_{11}w_{b,xxxx} + 2(D_{12} + D_{66})w_{b,xxyy} + D_{22} w_{b,yyyy} + \kappa A_{55}w_{s,xx} \\ &+ \kappa A_{44}w_{s,yy} = \rho I(\ddot{w}_{b,xx} + \ddot{w}_{b,yy}) \qquad (4\text{-}42) \end{aligned}$$

Equations (4-41) and (4-42) can also be expressed in the form

$$\begin{aligned} L_1w_b + L_2w_s &= \rho I \frac{\partial^2}{\partial t^2} \nabla^2 w_b \\ L_2w_s + q &= \rho h \frac{\partial^2}{\partial t^2} (w_b + w_s) \qquad (4\text{-}43) \end{aligned}$$

where

$$L_1 = D_{11} \frac{\partial^4}{\partial x^4} + 2(D_{12} + 2D_{66}) \frac{\partial^4}{\partial x^2 \partial y^2} + D_{22} \frac{\partial^4}{\partial y^4}$$

$$L_2 = \kappa A_{55} \frac{\partial^2}{\partial x^2} + \kappa A_{44} \frac{\partial^2}{\partial y^2}$$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

Applying Laplace transform to equations (4-43) yields

$$\begin{aligned} L_1\bar{w}_b + L_2\bar{w}_s &= \rho I\ s^2\nabla^2\bar{w}_b \\ L_2\bar{w}_s + \bar{q} &= \rho h\ s^2(\bar{w}_b + \bar{w}_s) \end{aligned} \tag{4-44}$$

where $\bar{w}_b$ and $\bar{w}_s$ are the transformed functions of w_b and w_s, respectively, and s is the Laplace transform parameter. Since the rotatory inertia is small, it is neglected in this study.

Solving Eqs. (4-44), we obtain

$$\left[(L_2 - L_1)\ \rho h\ s^2 + L_1 L_2 \right] \bar{w} = (-L_1 + L_2)\bar{q} \tag{4-45}$$

We expand the displacement w and the load q in terms of the shape functions w_{mn} (x,y) of the natural modes of the plate as

$$\begin{aligned} w &= \sum_m \sum_n B_{mn}(t) w_{mn}(x,y) \\ q &= \sum_m \sum_n q_{mn}(t) w_{mn}(x,y) \end{aligned} \tag{4-46}$$

For a simply-supported rectangular plate, the shape function for the (m,n) mode is given by

$$w_{mn}(x,y) = \sin \frac{m\pi x}{a} \quad \sin \frac{n\pi y}{b} \tag{4-47}$$

where a and b are the lateral dimensions of the plate.

Applying Laplace transform to Eq. (4-46) we obtain

$$\begin{aligned} \bar{w} &= \sum_m \sum_n \bar{B}_{mn}(s)\, w_{mn}(x,y) \\ \bar{q} &= \sum_m \sum_n \bar{q}_{mn}(s)\, w_{mn}(x,y) \end{aligned} \tag{4-48}$$

Substitution of Eqs. (4-48) and (4-47) into Eq. (4-45) leads to

$$\bar{B}_{mn}(s) = \frac{1}{\rho h} \frac{1}{s^2 + \omega_{mn}^2} \bar{q}_{mn}(s) \tag{4-49}$$

where

$$\omega_{mn}^2 = \frac{1}{h} \frac{C_{mn} E_{mn}}{C_{mn} + E_{mn}}$$

$$C_{mn} = D_{11} \left(\frac{m\pi}{a}\right)^4 + 2(D_{12} + 2D_{66})\left(\frac{m\pi}{a}\right)^2\left(\frac{n\pi}{b}\right)^2 + D_{22} \left(\frac{n\pi}{b}\right)^4$$

$$E_{mn} = \kappa A_{55} \left(\frac{m\pi}{a}\right)^2 + \kappa A_{44}\left(\frac{n\pi}{b}\right)^2 \tag{4-50}$$

The quantity ω_{mn} is the angular natural frequency for the (m,n) mode. If the transverse shear deformation is neglected (i.e. the classical plate theory), then

$$\omega_{mn}^2 = \frac{C_{mn}}{\rho h} \tag{4-51}$$

The solution for w can be obtained by applying inverse transform. We obtain

$$w = \frac{1}{\rho h} \sum_m \sum_n \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \int_0^t q_{mn}(\tau) \frac{\sin\omega_{mn}(t-\tau)}{\omega_{mn}} d\tau \tag{4-52}$$

where

$$q_{mn}(t) = \frac{4}{ab} \int_0^a \int_0^b q(x,y,t) \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \, dxdy \tag{4-53}$$

Consider a contact force given as a sine function, see Eq. (4-16), which is applied at the point (x_1, y_1). Then

$$q_{mn}(t) = \frac{4}{ab} \int_0^a \int_0^b q(x,y,t) \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \, dxdy$$

$$= \frac{4}{ab} F_{max} \sin\left(\frac{\pi t}{T}\right) \sin\frac{m\pi x_1}{a} \sin\frac{n\pi y_1}{b} \text{ for } t \leq T \tag{4-54}$$

Substitution of Eq. (4-54) into Eq. (4-52) yields

$$w(x,y,t) = \frac{4F_{max}}{\rho hab} \sum_m^\infty \sum_n^\infty \left[\left(\sin\frac{m\pi x_1}{a} \sin\frac{n\pi y_1}{b}\right)\left(\sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b}\right) \right.$$

$$\left. \frac{1}{\omega_{mn}^2} \frac{1}{1 - \left(\frac{\pi}{\omega_{mn} T}\right)^2} \left(\sin\frac{\pi}{T} t - \frac{\pi}{T\omega_{mn}} \sin\omega_{mn} t\right) \right] \tag{4-55}$$

for $0 \le t \le T$. For $x_1 = a/2$, $y_1 = b/2$, Eq. (4-55) becomes

$$w(x,y,t) = \frac{4F_{max}}{\rho hab} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left[(\sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}) \frac{1}{\omega_{mn}^2} \right.$$

$$\left. \frac{1}{1 - (\frac{\pi}{\omega_{mn} T})^2} (\sin \frac{\pi}{T} t - \frac{\pi}{T\omega_{mn}} \sin \omega_{mn} t) \right] \sin \frac{m\pi}{2} \sin \frac{n\pi}{2} \tag{4-56}$$

comparing Eq. (4-46) with (4-56) we find

$$B_{mn}(t) = \frac{4F_{max}}{\rho hab} \frac{1}{\omega_{mn}^2} \frac{1}{1 - (\frac{\pi}{\omega_{mn} T})^2} \times \sin \frac{m\pi}{2} \sin \frac{n\pi}{2} \times (\sin \frac{\pi}{T} t - \frac{\pi}{T\omega_{mn}} \sin \omega_{mn} t) \tag{4-57}$$

The kinetic energy in the plate at any time $t \le T$ is given by

$$K(t) = \frac{\rho h}{2} \int_0^a \int_0^b (\frac{\partial w}{\partial t})^2 \, dxdy \tag{4-58}$$

Substituting Eq. (4-57) into Eq. (4-46) and then into Eq. (4-58) we obtain

$$K(t) = \frac{\rho hab}{8} \sum_m \sum_n \dot{B}_{mn}^2(t) \tag{4-59}$$

By introducing the stiffness, K_{mn}, of the plate system for the (m, n) made, the total strain energy can be formally written as

$$U(t) = \frac{1}{2} \sum_m \sum_n K_{mn} B_{mn}^2(t) \tag{4-60}$$

Upon substitution of Eqs. (4-59) and (4-60) into the Lagrangian equation of motion we obtain

$$\frac{1}{4}\ \rho hab\ \ddot{B}_{mn}(t) + K_{mn}B_{mn}(t) = Q_{mn} \tag{4-61}$$

where Q_{mn} is the generalized force. From Eq. (4-61) we obtain the natural frequency ω_{mn} for the (m,n) made as

$$\omega_{mn}^2 = \frac{4}{\rho hab}\ K_{mn} \tag{4-62}$$

from which

$$K_{mn} = \frac{\rho hab}{4}\ \omega_{mn}^2 \tag{4-63}$$

Substituting Eqs. (4-63) and (4-57) into Eq. (4-60) we obtain

$$U\ (t) = \frac{2F_{max}^2}{\rho hab} \sum_m \sum_n \left[\frac{1}{\omega_{mn}} \ \frac{1}{1 - (\frac{\pi}{\omega_{mn}T})^2} \times \sin\frac{m\pi}{2}\ \sin\frac{n\pi}{2} \times \right.$$

$$\left. \times \left(\sin\frac{\pi}{T}\ t - \frac{\pi}{T\omega_{mn}}\ \sin\omega_{mn}t\right) \right]^2 \tag{4-64}$$

With Eqs. (4-60) and (4-64), the quantities U* and K* in the equivalent mass mode can be obtained. We have

$$U^*\,(T/2) = \frac{2}{\rho hab}\; f_3 \tag{4-65}$$

$$K^*\,(T/2) = \frac{2\pi^2}{\rho habT^2}\; g_3 \tag{4-66}$$

where

$$f_3 = \sum_m \sum_n \left[\frac{1}{\omega_{mn}} \; \frac{1}{1 - (\frac{\pi}{\omega_{mn} T})^2} \; (1 - \frac{\pi}{T\omega_{mn}} \sin \frac{\omega_{mn} T}{2}) \sin \frac{m\pi}{2} \sin \frac{n\pi}{2} \right]^2 \tag{4-67}$$

$$g_3 = \sum_m \sum_n \left[\frac{1}{\omega_{mn}^2} \; \frac{1}{1 - (\frac{\pi}{\omega_{mn} T})^2} \cos (\frac{\omega_{mn} T}{2}) \sin \frac{m\pi}{2} \; \sin \frac{n\pi}{2} \right]^2 \tag{4-68}$$

Karas [13] considered the impact of a steel ball of 2 cm in diameter on a simply-supported square steel plate with a=b=20 cm and h = 0.8 cm by using the classical plate theory. The impact velocity of the ball was 100 cm/sec. The contact force histories obtained by Karas and by using the equivalent mass model are shown in Fig. 4.8. It is seen that the equivalent mass model yields a good estimate of the maximum contact force and contact duration.

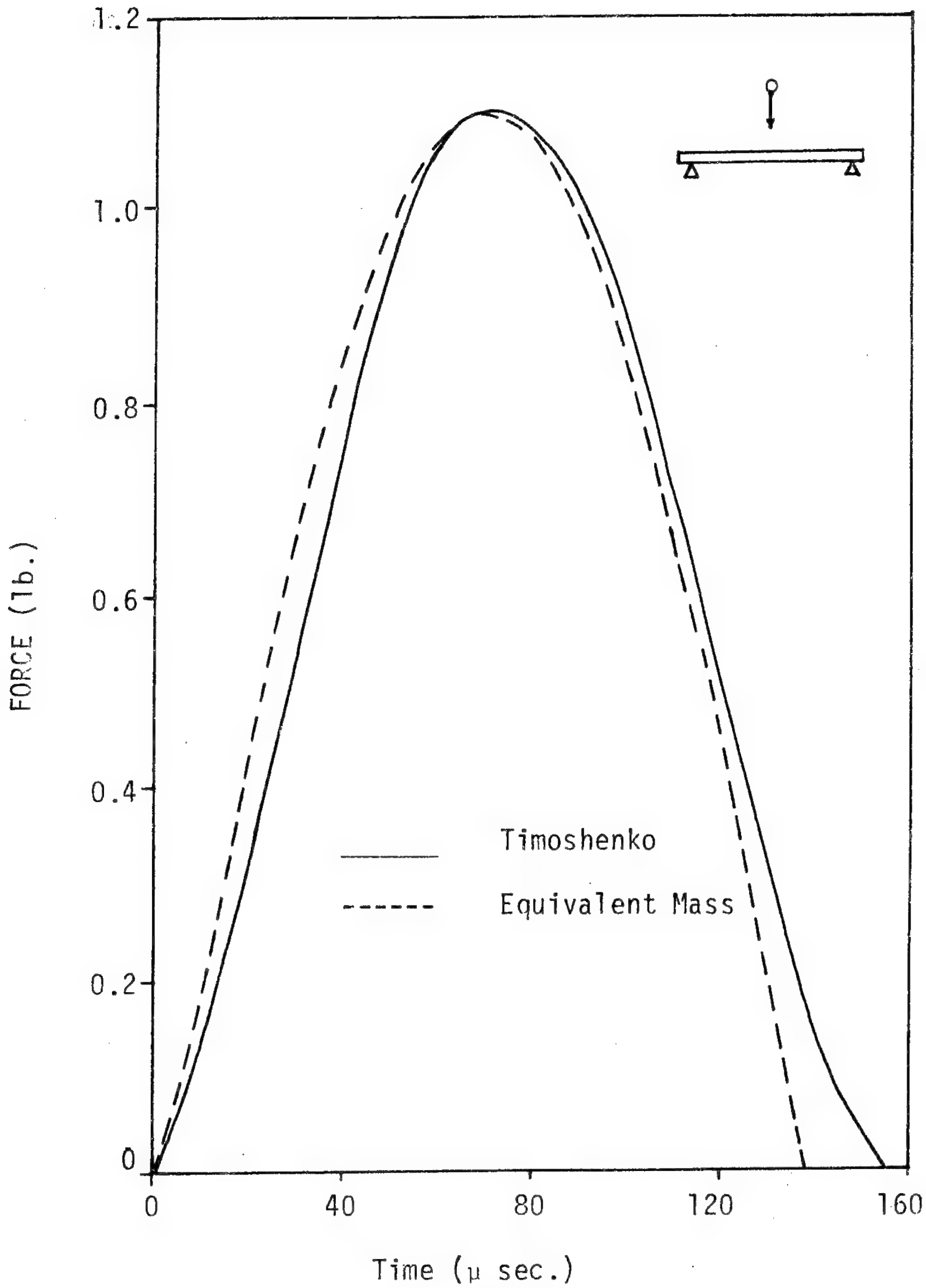

Fig. 4.1 Contact force history for the Timoshenko problem.

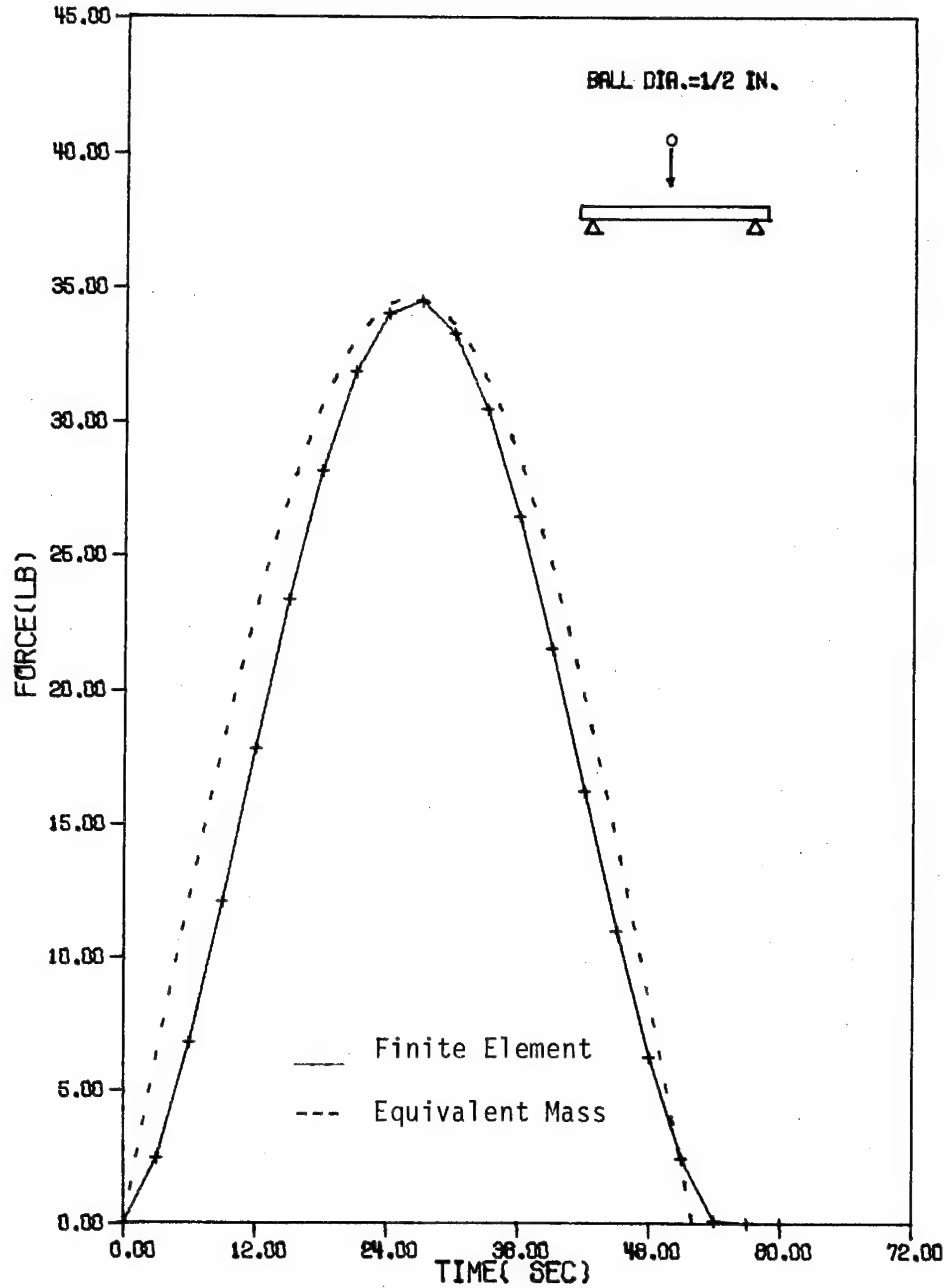

Fig. 4.2. Simply-supported steel beam (0.5"W x 0.5"D x 30"L) subjected to impact of a steel ball at 12 in/sec.

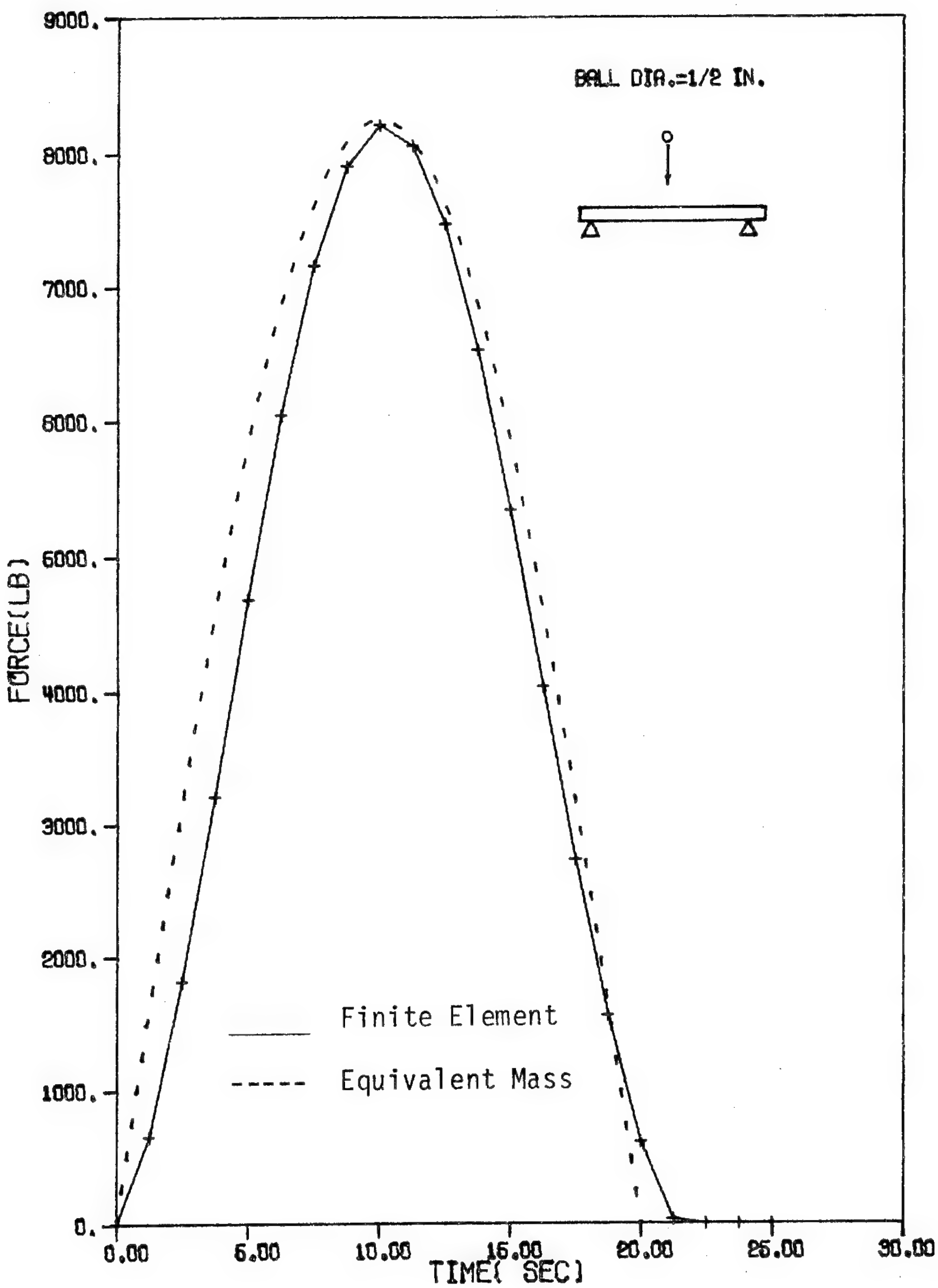

Fig. 4.3 Simply-supported steel beam (0.5"W x 0.5"D x 30"L) subjected to impact of a steel ball at 1200 in/sec.

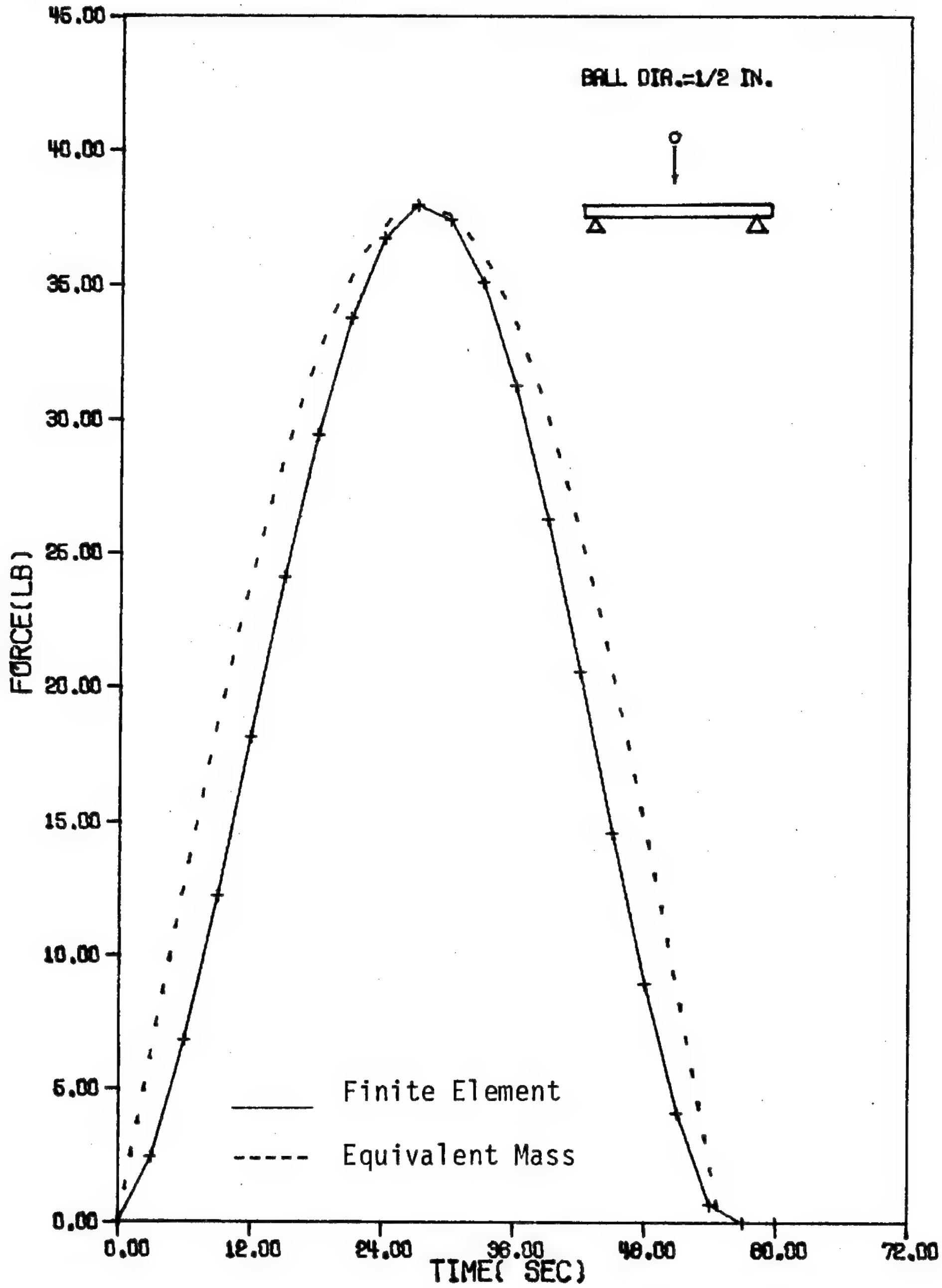

Fig. 4.4 Simply-supported steel beam (0.5"W x 3"D x 30"L) subjected to impact of a steel ball at 12 in/sec.

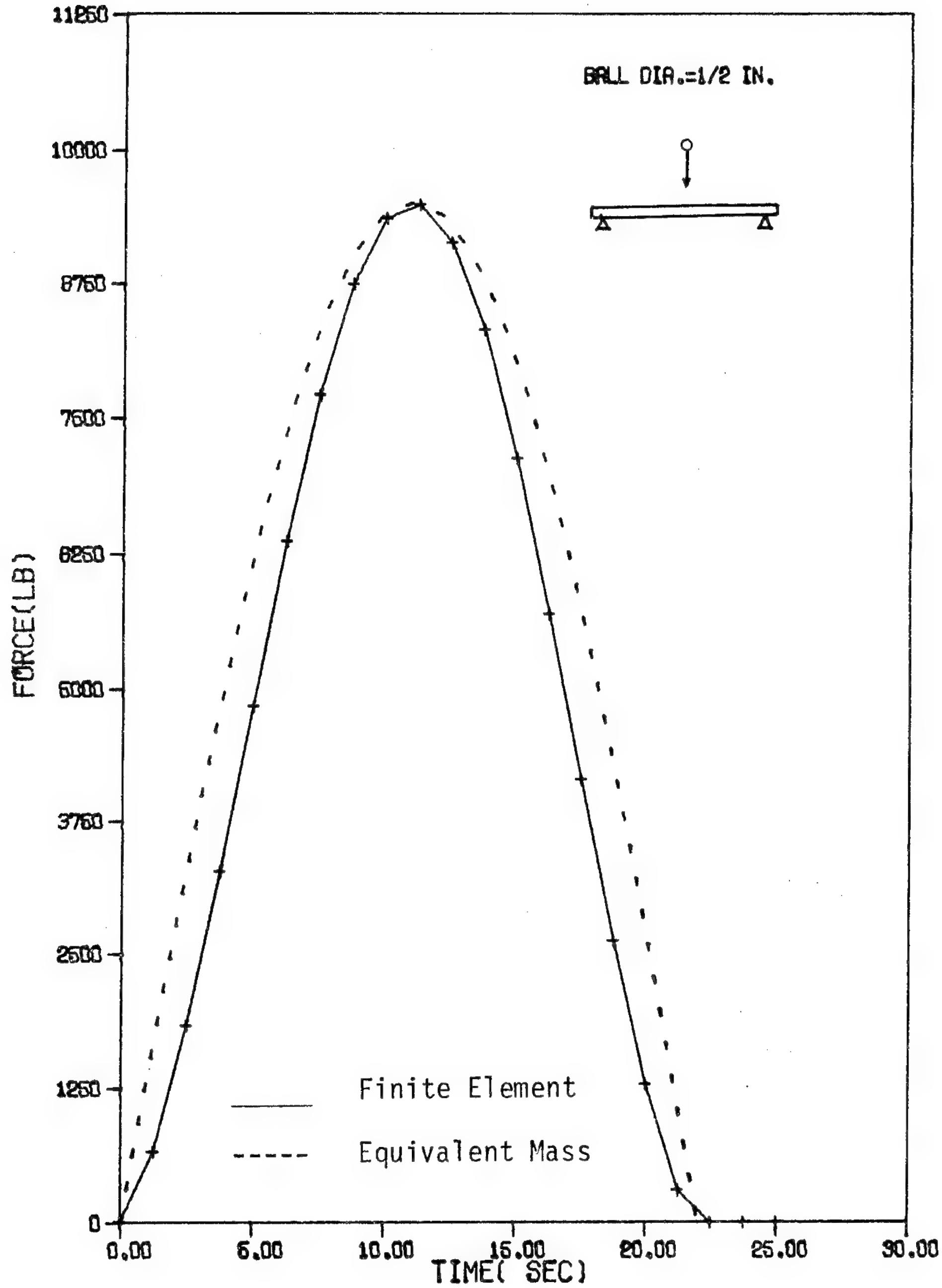

Fig. 4.5 Simply-supported steel beam (0.5"W x 3"D x 30"L) subjected to impact of a steel ball at 1200 in/sec.

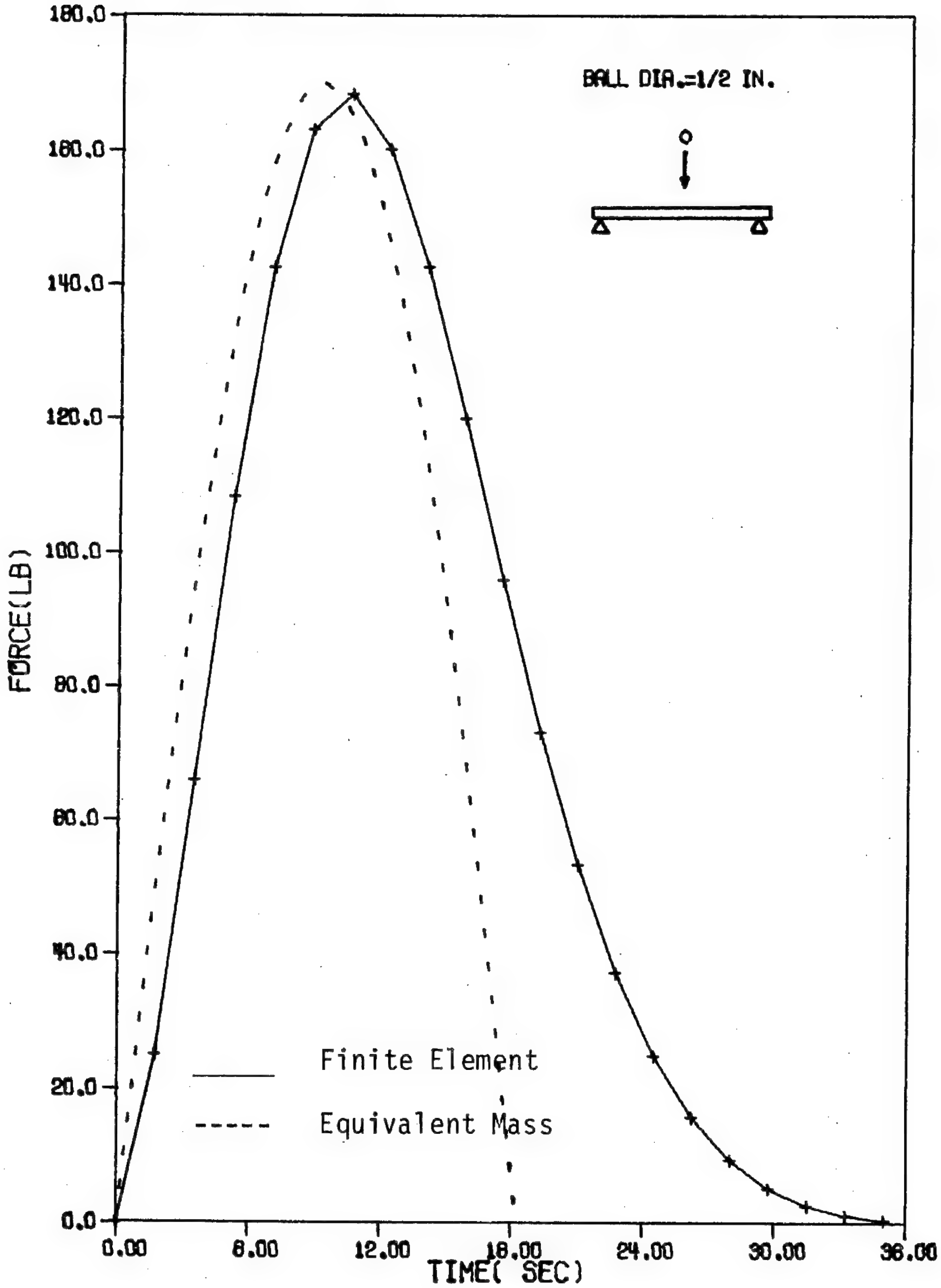

Fig. 4.6 Simply-supported steel beam (0.5"W x 0.08"D x 15"L) subjected to impact of a steel ball at 100 in/sec.

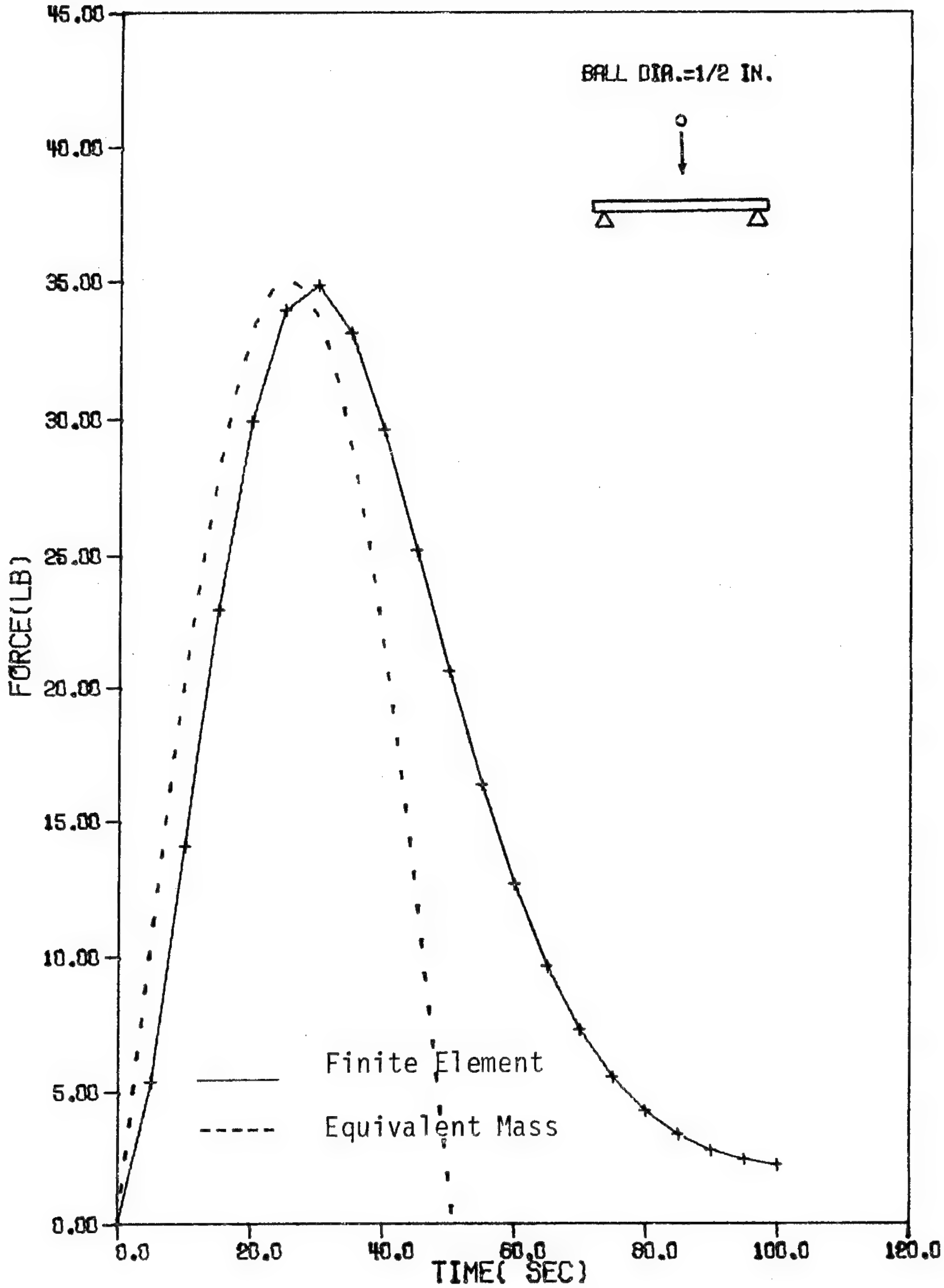

Fig. 4.7 Simply-supported graphite/epoxy beam (0.5"W x 0.08"D x 15"L) subjected to impact of a steel ball at 100 in/sec.

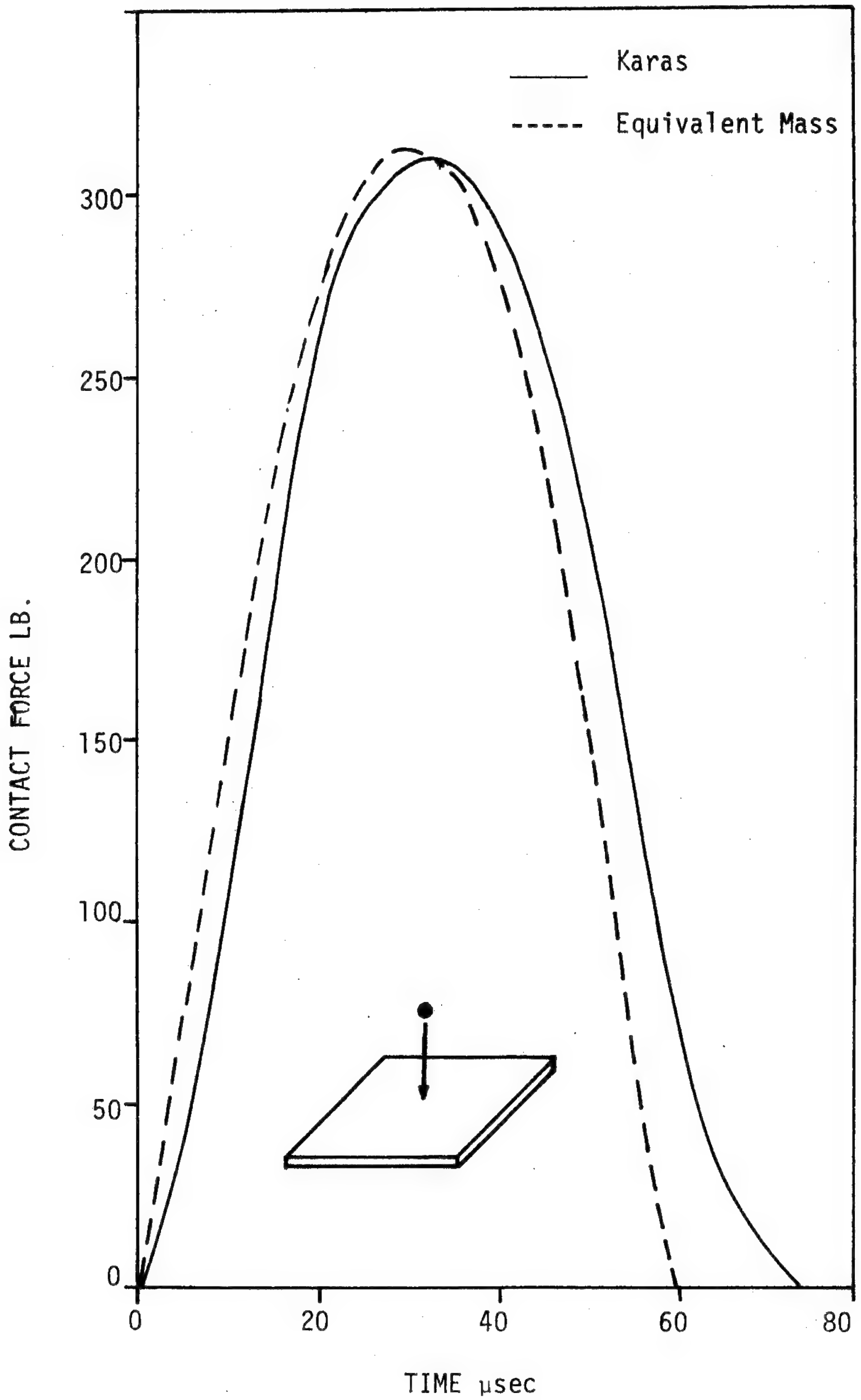

Fig. 4.8 Contact force history for a simply-supported steel plate (20 cm x 20 cm x 0.8 cm) subjected to impact of a steel ball (2 cm diameter) at 100 cm/sec.

5. Conclusions

Static indentation tests have been performed to determine the law of contact between a steel ball and two laminated composites, namely, glass/epoxy and graphite/epoxy. It has been found that the loading path followed very well the power law

$$F = k\,\alpha^{1.5}$$

where F is the contact force, k is a coefficient, and α is the indentation depth. Tests were conducted with beams clamped at two ends with various spans. The results indicated that the indentation law does not seem to depend on the span between the clamps. The experimental results have also revealed that both composites tested possessed a pronounced inelastic behavior even at very low contact force levels. The unloading paths from various loading points have been obtained experimentally and fitted into a power law for the computational purpose.

An efficient high order beam finite element has been employed together with the classical Hertzian contact law or the measured contact law for analyzing the impact response. The finite element program is capable of computing the contact force, contact duration, and all the dynamic responses in the laminated composite. A simple method for estimating the contact force and duration has been developed and shown to be quite accurate except for very thin beams.

6. References

[1] Hertz, H., "Über die Berührung fester elastischer Körper", Journal Reine Angle Math (Crelle), Vol. 92, 1881, p. 155.

[2] Willis, J.R., "Hertzian Contact of Anisotropic Bodies," Journal of Mechanics and Physics of Solids, Vol. 14, 1966, p. 163.

[3] Sun, C.T., "An Analytical Method for Evaluation of Impact Damage Energy of Laminated Composites," ASTM STP617, 1977, p. 427.

[4] Barnhart, K.E., and Goldsmith, W., "Stresses in Beams during Transverse Impact," J. Appl. Mech., Vol. 24, 1957, p. 440.

[5] Sun, C.T., and Huang, S.N., "Transverse Impact Problems by Higher Order Beam Finite Element," Journal of Computers and Structures, Vol. 5, pp. 297-303, 1975.

[6] Wilson, E.L. and Clough, R.W., "Dynamic Response by Step by Step Matrix Analysis," Symposium on Use of Computers in Civil Engineering, October 1962.

[7] Timoshenko, S.P., Theory of Elasticity, McGraw-Hill, New York, 1934.

[8] Goldsmith, W., Impact, Edward Arnold, London, 1960, p. 58.

[9] Timoshenko, S. "Zür Frage nach der wirkung eines Stosse auf einer Balken," Zaitschrift für Mathematik und Physik, Vol. 62, 1913, pp. 198-209.

[10] Whitney, J.M., and Pagano, N.J., "Shear Deformation in Heterogeneous Anisotropic Plates," J. Applied Mechanics, Vol. 37, 1970, pp. (031-1036.

[11] Sun, C.T., and Lai, R.Y.S., "Exact and Approximate Analysis of Transient Wave Propagation in an Anistropic Plate," AIAA Journal, Vol. 12, 1974, pp. 1415-1417.

[12] Mindlin, R.D., "Influence of Rotary Inertia and Shear on Flexural Vibrations of Isotropic, Elastic Plates," J. Applied Mechanics, Vol. 18, 1951, pp. 31-38.

[13] Karas, K., "Platten Unter Seitlichem Stoss," Ingenieur-Archiv, Vol. 10, 1939, pp. 237-250.

APPENDIX A

A COMPUTER PROGRAM FOR FINITE ELEMENT ANALYSIS OF THE TRANSVERSE IMPACT OF A BEAM

The following is a description of the input data required to analyze the transverse impact of a beam. The description is by card sections, and where applicable, the number of cards precedes the name. The arrangement of the cards is shown in Fig. A-1.

1. Heading Card(s) (I2, 10A7)

   One card is required for each problem.

   Cols. 1-2 Problem number (NPROB)

   3-72 Arbitrary problem identification (TITLE)

2. 1-Control Card (9I5)

   Cols. 1-5 Number of nodal points (NP)

   6-10 Number of elements (NE)

   11-15 Number of restrained boundary nodes (NB)

   16-20 Number of output printing cycles (NTM)

   21-25 Number of material types (NMAT)

   For isotropic materials, this number is limited to 24 plus one for the sphere. However, for a laminated composite, this number can only be two.

   26-30 Output printing frequency in $\frac{1}{10}$ μsec (NDIN)

   31-35 Beam material type (MATP)

   0 - if beam is isotropic

   1 - if beam is a laminated composite

   36-50 Number of nodal data cards (NDC)

   Explained later.

41-45 Control for print of input data (11)

0 - Input printed at beginning of first problem only.

1 - Input printed for each new problem.

Input for the printing scheme outlines the cycle and frequency at which the output is printed. The integer, NTM, indicates how many times output is printed after the sphere makes contact and the integer, NDIN, indicates how much time elapses between printing of the output. In addition, NDIM is measured in tenths of a microsecond. As an example, if one wishes to print output every 5 μsec for 10 cycles, then NDIN equals 50 (in $\frac{1}{10}$ μsec) and NTM=10. Observe that (NDON x NTRM)/10 yields the time at which computations stop, in this case its 50 μsec.

3. <u>1 - Dimension Card</u> (3F10.0)

Cols. 1-10 Beam thickness (TB)

11-20 Beam width (WB)

21-30 Sphere radius (R)

4. <u>1 - Nodal Impact Card</u> (I5,2F10.0)

Cols. 1-5 Impacted node (NQ)

6-15 Impact velocity (Q2)

16-25 Time increment (DT)

5. <u>Element Type Material Properties Card (s)</u> (I5,5F10.0)

1 card per material

Cols. 1-5 Material number (IMAT)

6-15 Longitudinal Young's modulus (ORT(N,1))

16-25 Transverse Young's modulus (ORT(N,2))

26-35 Shear modulus (ORT(N,3))

36-45 Poisson's ratio (ORT(N,4))

46-55 Mass density, ρ (ORT(N,5))

The last material card <u>must</u> contain the material properties of the impacting sphere. If the sphere and the beam possess identical material properties, then only one material card (NMAT = 1) is necessary.

6. <u>1 - Identation Law Card</u> (E10.3,2F10.0)

Cols. 1-10 Loading coefficient k (STF)

11-20 Permanent deformation α_0 (DISPEM)

21-30 Unloading power q (QP)

If the Hertzian law is used for loading, set STF = 0.0. If elastic unloading is followed, then set DISPEM = 0.0 and the input for QP will be ignored.

7. <u>Nodal Data Card(s)</u> (2I5, 2F10.0, I5)

1 card is required for each set of identical elements.

Cols. 1-5 Beginning node in the set (ND1)

6-10 Final node in the set (ND2)

11-20 x-position of beginning node (X1)

21-30 x-position of final node (X2)

31-35 Element material type of set (IMT)

This input provides information for the automatic element generator in the program. Given the above information for each set of identical elements, the program computes the x-position of each node and assigns each element a material type and the Ith and Jth nodes. The number of these cards is equal to NDC, which is input on the control card.

NOTE: Node 1 must begin at position x = 0.

8. <u>Boundary Conditions Card(s)</u> (2I5)

1 card per restrained node

Cols. 1-5 Restrained node (NBC)

6-10 Boundary condition code (NFIX)

The boundary condition code is an integer containing three digits.

Each digit in the code is either 1-restrained or 0-free. The ones digit controls the curvature, the tens digit controls the slope, and the hundreds digit controls the displacement. As an example, if one node was clamped, then the displacement and slope are zero and the curvature is nonzero, or

$$v = 0 \quad \theta = 0 \quad \kappa \neq 0 \qquad \text{therefore} \qquad \text{Code } 110$$

NOTE: Boundary conditions may be specified at any node with any code.

8. Number of layers in laminate (15) (MLAYER)
9. Laminate data (I5, F5.0, F10.0)

   1 card per layer.

   Cols. 1-5 Layer number (L)

   6-10 Fiber orientation (TH)

   11-20 Layer thickness (TK)

If a laminated composite beam is to be examined under impact, two major alterations in the program must be made. This program provides for both, with the proper indication on the control card (MATP = 1). From the laminate data given, an equivalent bending rigidity is computed, or D_{11} = EI. In addition, the contact coefficient in the Hertzian Contact Law is also computed differently for composite beams.

NOTE: If an isotropic beam is used, skip Cards 8 and 9.

10. Termination Card

<u>EXAMPLE 1</u>

Consider the impact of a steel sphere on a steel cantilever beam. The dimensions of the beam are 0.5" W x 0.08" D x 15" L and the sphere has a diameter of 0.5" in. The initial velocity of the sphere is 100 in/sec., with the point of impact located at the mid-point. Numerical solutions are to be obtained up to 100 μsec by using 30 finite elements.

The material constants used in this computation are $E = 30 \times 10^6$psi, $\nu = 0.25$ and $\rho = 0.00880$ slug/in^3(0.000733 lb - sec^2/in^4). Note that the value of ρ in slug should be divided by 12 if length is measured in inches.

The sample inputs and outputs for Example 1 and Example 2 are listed following Fig. A-1. The results for the contact force, the displacement of the sphere and the deflection of the beam at the impact point are shown in Fig. A-2. The displacement profiles of the beam at the impact point are shown in Fig. A-2. The displacement profiles of the beam at various times are presented in Fig. A-3.

EXAMPLE 2

This example is identical to the previous example except that the beam is now a laminated composite which consists of 16 layers of graphite/epoxy composite. The ply-thickness is 0.005" and the lay-up is $(0/90/0/90)_{2s}$. The material constants are

$$E_{11} = 30 \times 10^6 \text{ psi} \qquad E_{22} = 0.75 \times 10^6 \text{ psi} \qquad G_{12} = 0.4 \times 10^6 \text{ psi}$$

$$\nu_{12} = 0.25 \quad , \quad \rho = 0.00178 \text{ slug/in}^3 (0.000148 \text{ lb - sec}^2/\text{in}^4)$$

The corresponding results are shown in Figs. A-4 and A-5.

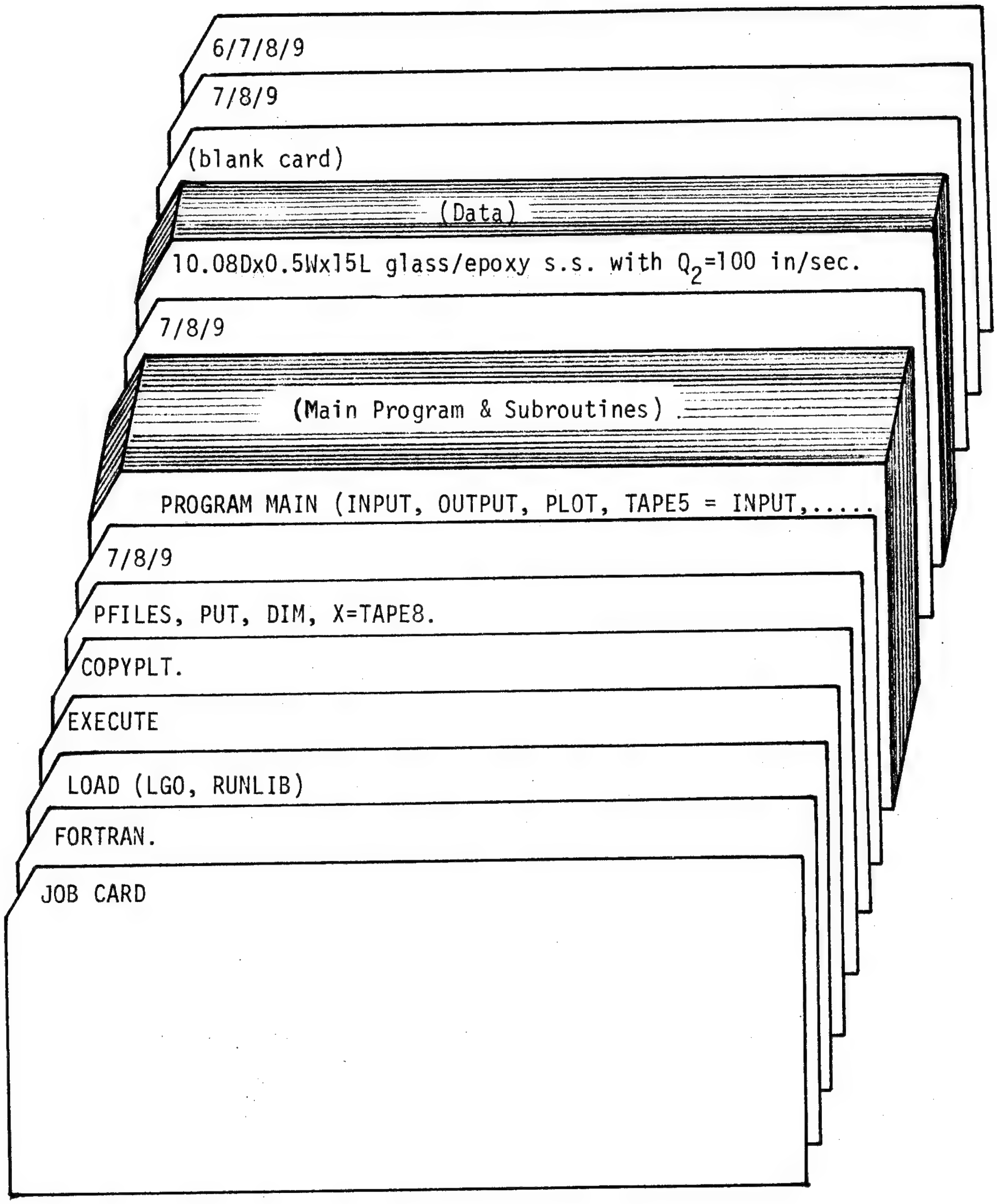

Fig. A-1 Deck set-up

Sample Input for Example 1

```
 1 0.08Dx0.5wx15L IS0. CANTILEVER WITH Q2=100 IN/SEC.
   31   30    1   20    1   50    0    1    0
     0.080     0.500     0.250
   16     100.0 3.500E-08
    1 30000000. 30000000. 11500000.  0.30       0.000733

    1   31    0.0    15.0      1
   31  110

7/8/9
6/7/8/9
```

*A standard card form, IBM electro 888157, is available for punching statements from this form

Sample Output for Example 1

```
 1  0.08DX0.5WX15L ISO. CANTILEVER WITH Q2=100 IN/SEC.

NODAL POINTS               31
ELEMENTS                   30
BOUNDARY CONDITIONS         1
OUTPUT LIMIT             1000
DEGREES OF FREEDOM          3
MATERIALS                   1

BEAM THICKNESS              .080
BEAM WIDTH                  .500
SPHERE DENSITY              .000733
SPHERE RADIUS               .250

IMPACT NODE              16
IMPACT VELOCITY            100.0
INTEGRATION TIME INCREMENT( X E-06 SEC) 3.500E-08

 MATERIAL PROPERTIES
MAT. NO.        E1            E2             G12          V12          RHO

   1      30000000.0    30000000.0    11500000.0       .300      .000733

PERMANENT DEFORMATION(IN)        *.000000

 NODAL POINTS
                X          Y
        1     0.000      0.000
        2      .500      0.000
        3     1.000      0.000
        4     1.500      0.000
        5     2.000      0.000
        6     2.500      0.000
        7     3.000      0.000
        8     3.500      0.000
        9     4.000      0.000
       10     4.500      0.000
       11     5.000      0.000
       12     5.500      0.000
       13     6.000      0.000
       14     6.500      0.000
       15     7.000      0.000
       16     7.500      0.000
       17     8.000      0.000
       18     8.500      0.000
       19     9.000      0.000
       20     9.500      0.000
       21    10.000      0.000
       22    10.500      0.000
       23    11.000      0.000
       24    11.500      0.000
       25    12.000      0.000
       26    12.500      0.000
       27    13.000      0.000
       28    13.500      0.000
       29    14.000      0.000
       30    14.500      0.000
       31    15.000      0.000
```

ELEMENTS

	I	J	K		MAT
1	1	2	0	0	1
2	2	3	0	0	1
3	3	4	0	0	1
4	4	5	0	0	1
5	5	6	0	0	1
6	6	7	0	0	1
7	7	8	0	0	1
8	8	9	0	0	1
9	9	10	0	0	1
10	10	11	0	0	1
11	11	12	0	0	1
12	12	13	0	0	1
13	13	14	0	0	1
14	14	15	0	0	1
15	15	16	0	0	1
16	16	17	0	0	1
17	17	18	0	0	1
18	18	19	0	0	1
19	19	20	0	0	1
20	20	21	0	0	1
21	21	22	0	0	1
22	22	23	0	0	1
23	23	24	0	0	1
24	24	25	0	0	1
25	25	26	0	0	1
26	26	27	0	0	1
27	27	28	0	0	1
28	28	29	0	0	1
29	29	30	0	0	1
30	30	31	0	0	1

BOUNDARY CONDITIONS

31 110

PRINTING SCHEME

1. REPORT OUTPUT EVERY 5.00 MSEC
2. TERMINATE OUTPUT AT100.00 MSEC

TYPICAL STIFNESS MATRIX OF AN ELEMENT

8.777E+04	2.194E+04	5.486E+02	-8.777E+04	2.194E+04	-5.486E+02
2.194E+04	7.022E+03	2.011E+02	-2.194E+04	3.950E+03	-7.314E+01
5.496E+02	2.011E+02	2.743E+01	-5.486E+02	7.314E+01	4.571E+00
-8.777E+04	-2.194E+04	-5.486E+02	8.777E+04	-2.194E+04	5.486E+02
2.194E+04	3.950E+03	7.314E+01	-2.194E+04	7.022E+03	-2.011E+02
-5.486E+02	-7.314E+01	4.571E+00	5.486E+02	-2.011E+02	2.743E+01

TYPICAL MASS MATRIX OF AN ELEMENT

5.743E-06	4.934E-07	1.858E-08	1.587E-06	-2.396E-07	1.197E-08
4.934E-07	5.500E-08	2.281E-09	2.396E-07	-3.517E-08	1.719E-09
1.858E-08	2.281E-09	9.916E-11	1.197E-08	-1.719E-09	8.263E-11
1.587E-06	2.396E-07	1.197E-08	5.743E-06	-4.934E-07	1.856E-08
-2.396E-07	-3.517E-08	-1.719E-09	-4.934E-07	5.500E-08	-2.281E-09
1.197E-08	1.719E-09	8.263E-11	1.858E-08	-2.281E-09	9.916E-11

0.08DX0.5WX15L ISO. CANTILEVER WITH Q2=100 IN/SEC.

TIME ELAPSED(MSEC)	10.500
FORCE(LB)	1.684E+02
MASS DISPLACEMENT(IN)	9.747E-04
MASS VELOCITY(IN/SEC)	7.849E+01
MASS ACCEL.(IN/SEC2)	-3.509E+06
INDENTATION(IN)	6.197E-04

NODE	DISP	STRAIN-XX	STRAIN-YY	STRESS-XX
1	1.670E-11	-9.892E-11	2.968E-11	-2.968E-03
2	-2.051E-11	5.506E-10	-1.652E-10	1.652E-02
3	3.951E-11	-9.258E-10	2.777E-10	-2.777E-02
4	-3.684E-11	3.027E-10	-9.081E-11	9.081E-03
5	-1.222E-10	5.425E-09	-1.628E-09	1.628E-01
6	8.400E-10	-2.698E-08	8.093E-09	-8.093E-01
7	-2.888E-09	8.059E-08	-2.418E-08	2.418E+00
8	6.521E-09	-1.550E-07	4.650E-08	-4.650E+00
9	-6.891E-09	8.597E-08	-2.579E-08	2.579E+00
10	-1.677E-08	6.471E-07	-1.941E-07	1.941E+01
11	1.002E-07	-2.525E-06	7.574E-07	-7.574E+01
12	-1.803E-07	2.561E-06	-7.683E-07	7.683E+01
13	-4.412E-07	1.034E-05	-3.101E-06	3.101E+02
14	-3.972E-07	2.215E-05	-6.645E-06	6.645E+02
15	-3.358E-05	1.188E-04	-3.565E-05	3.565E+03
16	3.578E-04	-7.110E-04	2.133E-04	-2.133E+04
17	-3.358E-05	1.188E-04	-3.565E-05	3.565E+03
18	-3.972E-07	2.215E-05	-6.645E-06	6.645E+02
19	-4.412E-07	1.034E-05	-3.101E-06	3.101E+02
20	-1.803E-07	2.561E-06	-7.683E-07	7.683E+01
21	1.002E-07	-2.525E-06	7.574E-07	-7.574E+01
22	-1.677E-08	6.471E-07	-1.941E-07	1.941E+01
23	-6.891E-09	8.597E-08	-2.579E-08	2.579E+00
24	6.521E-09	-1.550E-07	4.650E-08	-4.650E+00
25	-2.888E-09	8.059E-08	-2.418E-08	2.418E+00
26	8.400E-10	-2.698E-08	8.093E-09	-8.093E-01
27	-1.221E-10	5.424E-09	-1.627E-09	1.627E-01
28	-3.697E-11	3.067E-10	-9.201E-11	9.201E-03
29	3.985E-11	-9.352E-10	2.806E-10	-2.806E-02
30	-2.116E-11	5.698E-10	-1.709E-10	1.709E-02
31	1.306E-21	-2.755E-10	8.264E-11	-8.264E-03

0.08DX0.5WX15L ISO. CANTILEVER WITH Q2=100 IN/SEC.

TIME ELAPSED(MSEC)	35.000
FORCE(LB)	3.223E-01
MASS DISPLACEMENT(IN)	2.332E-03
MASS VELOCITY(IN/SEC)	4.839E+01
MASS ACCEL.(IN/SEC2)	-6.718E+03
INDENTATION(IN)	1.139E-05

NODE	DISP	STRAIN-XX	STRAIN-YY	STRESS-XX
1	-1.639E-08	1.939E-07	-5.817E-08	5.817E+00
2	-4.074E-08	1.315E-06	-3.944E-07	3.944E+01
3	1.731E-07	-3.163E-06	9.490E-07	-9.490E+01
4	8.057E-08	-1.543E-06	4.630E-07	-4.630E+01
5	-1.623E-07	1.457E-06	-4.371E-07	4.371E+01
6	-5.161E-07	6.356E-06	-1.907E-06	1.907E+02
7	-8.232E-07	8.349E-06	-2.505E-06	2.505E+02
8	-1.621E-06	1.321E-05	-3.964E-06	3.964E+02
9	-3.188E-06	1.493E-05	-4.478E-06	4.478E+02
10	-1.613E-07	-1.602E-05	4.806E-06	-4.806E+02
11	2.248E-05	-5.990E-05	1.797E-05	-1.797E+03
12	-4.545E-05	1.443E-04	-4.329E-05	4.329E+03
13	9.286E-05	-2.545E-04	7.636E-05	-7.636E+03
14	-3.946E-04	4.636E-04	-1.391E-04	1.391E+04
15	1.079E-03	-6.524E-05	1.957E-05	-1.957E+03
16	2.322E-03	-4.584E-04	1.375E-04	-1.375E+04
17	1.079E-03	-6.524E-05	1.957E-05	-1.957E+03
18	-3.946E-04	4.636E-04	-1.391E-04	1.391E+04
19	9.286E-05	-2.545E-04	7.636E-05	-7.636E+03
20	-4.545E-05	1.443E-04	-4.329E-05	4.329E+03
21	2.248E-05	-5.990E-05	1.797E-05	-1.797E+03
22	-1.612E-07	-1.602E-05	4.807E-06	-4.807E+02
23	-3.188E-06	1.493E-05	-4.478E-06	4.478E+02
24	-1.621E-06	1.322E-05	-3.966E-06	3.966E+02
25	-8.225E-07	8.333E-06	-2.500E-06	2.500E+02
26	-5.171E-07	6.379E-06	-1.914E-06	1.914E+02
27	-1.620E-07	1.454E-06	-4.362E-07	4.362E+01
28	8.263E-08	-1.598E-06	4.793E-07	-4.793E+01
29	1.684E-07	-3.057E-06	9.171E-07	-9.171E+01
30	-3.759E-08	1.254E-06	-3.762E-07	3.762E+01
31	-2.132E-18	5.192E-08	-1.558E-08	1.558E+00

Sample Input for Example 2

```
 10.008DX0.5WX15L COMP. CANTILEVER WITH Q2=100 IN/SEC.
   31   30    1   20    2   50    1    1    0
     0.080     0.500     0.250
   16     100.0 1.000E-07
    1 30000000.   750000.   400000. 0.25       0.000148
    2 30000000.300000000. 11500000. 0.30       0.000733

    1   31    0.0       15.0      1
   31  110
   16
    1   0.     0.005
    2  90.     0.005
    3   0.     0.005
    4  90.     0.005
    5   0      0.005
    6  90.     0.005
    7   0.     0.005
    8  90.     0.005
    9  90.     0.005
   10   0.     0.005
   11  90.     0.005
   12   0.     0.005
   13  90.     0.005
   14   0.     0.005
   15  90.     0.005
   16   0.     0.005

7/8/9
6/7/8/9
```

Sample Output for Example 2

```
 1  0.08DX0.5WX15L COMP. CANTILEVER WITH Q2=100 IN/SEC

NODAL POINTS              31
ELEMENTS                  30
BOUNDARY CONDITIONS        1
OUTPUT LIMIT            1000
DEGREES OF FREEDOM         3
MATERIALS                  2

BEAM THICKNESS                 .080
BEAM WIDTH                     .500
SPHERE DENSITY                 .000733
SPHERE RADIUS                  .250

IMPACT NODE                 16
IMPACT VELOCITY              100.0
INTEGRATION TIME INCREMENT( X E-06 SEC) 1.000E-07

 MATERIAL PROPERTIES
MAT. NO.      E1            E2          G12          V12        RHO

  1        30000000.0    750000.0     400000.0      .250     .000148

  2        30000000.0  30000000.0   11500000.0      .300     .000733

PERMANENT DEFORMATION(IN)       *.000000

ABD MATRIX
  1.232E+06  1.502E+04 -5.912E-39 -2.910E-11  3.183E-12 -9.788E-55
  1.502E+04  1.232E+06 -1.994E-09  3.183E-12  3.402E-10 -4.653E-25
 -5.912E-39 -1.994E-09  3.200E+04 -9.788E-55 -4.653E-25  4.547E-12
 -2.910E-11  3.183E-12 -9.788E-55  7.742E+02  8.013E+00 -2.562E-42
  3.183E-12  3.402E-10 -4.653E-25  8.013E+00  5.398E+02 -8.639E-13
 -9.788E-55 -4.653E-25  4.547E-12 -2.562E-42 -8.639E-13  1.707E+01

 NODAL POINTS
              X          Y
       1    0.000      0.000
       2     .500      0.000
       3    1.000      0.000
       4    1.500      0.000
       5    2.000      0.000
       6    2.500      0.000
       7    3.000      0.000
       8    3.500      0.000
       9    4.000      0.000
      10    4.500      0.000
      11    5.000      0.000
      12    5.500      0.000
      13    6.000      0.000
      14    6.500      0.000
      15    7.000      0.000
      16    7.500      0.000
      17    8.000      0.000
      18    8.500      0.000
      19    9.000      0.000
      20    9.500      0.000
      21   10.000      0.000
      22   10.500      0.000
      23   11.000      0.000
      24   11.500      0.000
      25   12.000      0.000
      26   12.500      0.000
      27   13.000      0.000
      28   13.500      0.000
      29   14.000      0.000
      30   14.500      0.000
      31   15.000      0.000
```

ELEMENTS

	I	J	K		MAT
1	1	2	0	0	1
2	2	3	0	0	1
3	3	4	0	0	1
4	4	5	0	0	1
5	5	6	0	0	1
6	6	7	0	0	1
7	7	8	0	0	1
8	8	9	0	0	1
9	9	10	0	0	1
10	10	11	0	0	1
11	11	12	0	0	1
12	12	13	0	0	1
13	13	14	0	0	1
14	14	15	0	0	1
15	15	16	0	0	1
16	16	17	0	0	1
17	17	18	0	0	1
18	18	19	0	0	1
19	19	20	0	0	1
20	20	21	0	0	1
21	21	22	0	0	1
22	22	23	0	0	1
23	23	24	0	0	1
24	24	25	0	0	1
25	25	26	0	0	1
26	26	27	0	0	1
27	27	28	0	0	1
28	28	29	0	0	1
29	29	30	0	0	1
30	30	31	0	0	1

BOUNDARY CONDITIONS
31 110

PRINTING SCHEME
1. REPORT OUTPUT EVERY 5.00 MSEC
2. TERMINATE OUTPUT AT100.00 MSEC

TYPICAL STIFNESS MATRIX OF AN ELEMENT

1.062E+05	2.654E+04	6.636E+02	-1.062E+05	2.654E+04	-6.636E+02
2.654E+04	8.494E+03	2.433E+02	-2.654E+04	4.778E+03	-8.848E+01
6.636E+02	2.433E+02	3.318E+01	-6.636E+02	8.848E+01	5.530E+00
-1.062E+05	-2.654E+04	-6.636E+02	1.062E+05	-2.654E+04	6.636E+02
2.654E+04	4.778E+03	8.848E+01	-2.654E+04	8.494E+03	-2.433E+02
-6.636E+02	-8.848E+01	5.530E+00	6.636E+02	-2.433E+02	3.318E+01

TYPICAL MASS MATRIX OF AN ELEMENT

1.160E-06	9.963E-08	3.751E-09	3.203E-07	-4.837E-08	2.416E-09
9.963E-08	1.111E-08	4.605E-10	4.837E-08	-7.101E-09	3.470E-10
3.751E-09	4.605E-10	2.002E-11	2.416E-09	-3.470E-10	1.668E-11
3.203E-07	4.837E-08	2.416E-09	1.160E-06	-9.963E-08	3.751E-09
-4.837E-08	-7.101E-09	-3.470E-10	-9.963E-08	1.111E-08	-4.605E-10
2.416E-09	3.470E-10	1.668E-11	3.751E-09	-4.605E-10	2.002E-11

0.08DX0.5WX15L COMP. CANTILEVER WITH Q2=100 IN/SEC

TIME ELAPSED(MSEC)	20.000
FORCE(LB)	2.989E+01
MASS DISPLACEMENT(IN)	1.963E-03
MASS VELOCITY(IN/SEC)	9.407E+01
MASS ACCEL.(IN/SEC2)	-6.230E+05
INDENTATION(IN)	1.557E-03

NODE	DISP	STRAIN-XX	STRAIN-YY	STRESS-XX
1	4.354E-09	-1.203E-08	3.008E-09	-3.610E-01
2	1.211E-09	-1.140E-08	2.850E-09	-3.420E-01
3	1.594E-09	-1.941E-08	4.853E-09	-5.824E-01
4	4.063E-09	-5.353E-08	1.338E-08	-1.606E+00
5	1.002E-08	-1.048E-07	2.619E-08	-3.143E+00
6	2.165E-08	-1.112E-07	2.780E-08	-3.336E+00
7	2.325E-08	-4.028E-08	1.007E-08	-1.209E+00
8	-7.712E-08	3.099E-07	-7.748E-08	9.297E+00
9	-2.602E-08	-1.633E-07	4.081E-08	-4.898E+00
10	3.666E-07	-2.982E-07	7.454E-08	-8.945E+00
11	-1.191E-06	8.802E-07	-2.201E-07	2.641E+01
12	2.941E-06	-8.390E-08	2.098E-08	-2.517E+00
13	-1.862E-06	-9.902E-06	2.476E-06	-2.971E+02
14	-3.526E-05	3.434E-05	-8.586E-06	1.030E+03
15	1.289E-04	3.957E-05	-9.892E-06	1.187E+03
16	4.115E-04	-1.934E-04	4.835E-05	-5.802E+03
17	1.289E-04	3.957E-05	-9.892E-06	1.187E+03
18	-3.526E-05	3.434E-05	-8.586E-06	1.030E+03
19	-1.862E-06	-9.902E-06	2.476E-06	-2.971E+02
20	2.941E-06	-8.370E-08	2.092E-08	-2.511E+00
21	-1.191E-06	8.798E-07	-2.199E-07	2.639E+01
22	3.666E-07	-2.978E-07	7.446E-08	-8.935E+00
23	-2.603E-08	-1.629E-07	4.073E-08	-4.888E+00
24	-7.707E-08	3.088E-07	-7.721E-08	9.265E+00
25	2.321E-08	-3.940E-08	9.850E-09	-1.182E+00
26	2.164E-08	-1.112E-07	2.779E-08	-3.335E+00
27	1.003E-08	-1.050E-07	2.624E-08	-3.149E+00
28	4.071E-09	-5.328E-08	1.332E-08	-1.599E+00
29	1.696E-09	-2.162E-08	5.406E-09	-6.487E-01
30	1.136E-09	-9.681E-09	2.420E-09	-2.904E-01
31	2.507E-18	-3.823E-08	9.556E-09	-1.147E+00

0.08DX0.5WX15L COMP. CANTILEVER WITH Q2=100 IN/SEC

TIME ELAPSED(MSEC)	90.000
FORCE(LB)	2.785E+00
MASS DISPLACEMENT(IN)	7.288E-03
MASS VELOCITY(IN/SEC)	6.770E+01
MASS ACCEL.(IN/SEC2)	-5.805E+04
INDENTATION(IN)	3.264E-04

NODE	DISP	STRAIN-XX	STRAIN-YY	STRESS-XX
1	3.708E-05	-2.198E-08	5.496E-09	-6.595E-01
2	-3.087E-05	2.453E-05	-6.132E-06	7.358E+02
3	2.007E-05	-1.937E-06	4.844E-07	-5.812E+01
4	5.413E-05	-3.431E-05	8.577E-06	-1.029E+03
5	-7.656E-05	1.602E-05	-4.004E-06	4.805E+02
6	-1.137E-04	5.467E-05	-1.367E-05	1.640E+03
7	1.365E-04	-3.582E-06	8.954E-07	-1.074E+02
8	3.434E-04	-9.696E-05	2.424E-05	-2.909E+03
9	5.487E-06	-7.986E-05	1.996E-05	-2.396E+03
10	-7.572E-04	6.927E-05	-1.732E-05	2.078E+03
11	-1.100E-03	1.993E-04	-4.982E-05	5.978E+03
12	-2.664E-04	1.998E-04	-4.995E-05	5.994E+03
13	1.748E-03	7.519E-05	-1.880E-05	2.256E+03
14	4.217E-03	-8.072E-05	2.018E-05	-2.422E+03
15	6.202E-03	-1.999E-04	4.998E-05	-5.998E+03
16	6.969E-03	-2.614E-04	6.536E-05	-7.843E+03
17	6.202E-03	-1.999E-04	4.998E-05	-5.998E+03
18	4.217E-03	-8.072E-05	2.018E-05	-2.422E+03
19	1.748E-03	7.519E-05	-1.880E-05	2.256E+03
20	-2.664E-04	1.998E-04	-4.995E-05	5.994E+03
21	-1.100E-03	1.993E-04	-4.982E-05	5.978E+03
22	-7.572E-04	6.927E-05	-1.732E-05	2.078E+03
23	5.487E-06	-7.986E-05	1.997E-05	-2.396E+03
24	3.434E-04	-9.696E-05	2.424E-05	-2.909E+03
25	1.365E-04	-3.579E-06	8.949E-07	-1.074E+02
26	-1.137E-04	5.467E-05	-1.367E-05	1.640E+03
27	-7.654E-05	1.603E-05	-4.007E-06	4.808E+02
28	5.425E-05	-3.428E-05	8.571E-06	-1.029E+03
29	2.033E-05	-2.156E-06	5.389E-07	-6.467E+01
30	-3.311E-05	2.110E-05	-5.275E-06	6.330E+02
31	-3.552E-15	-3.097E-05	7.743E-06	-9.291E+02

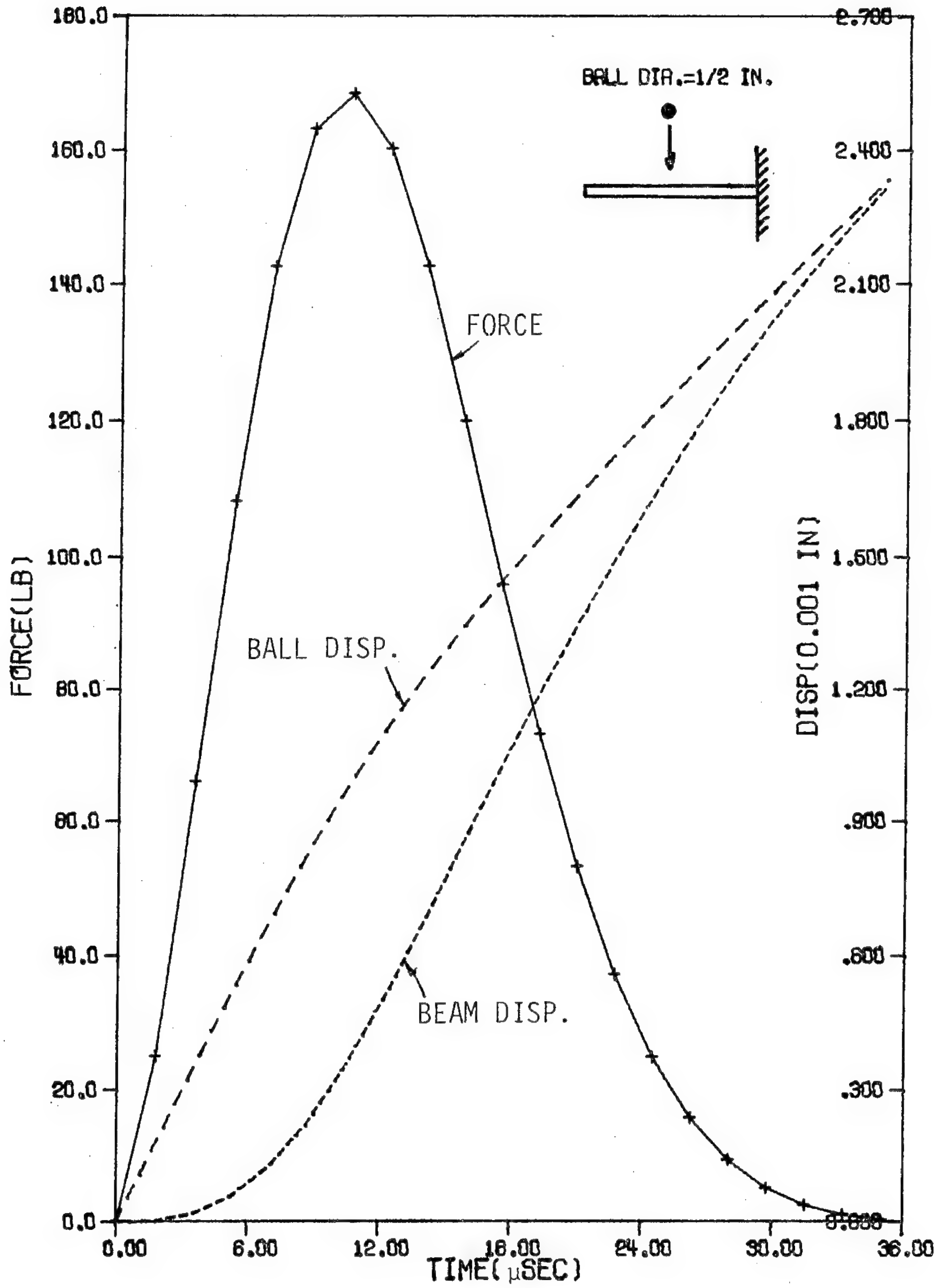

Fig. A-2 Response of a cantilever steel beam (0.5"W x 0.08"D x 15"L) subjected to impact of a steel ball at 100 in./sec.

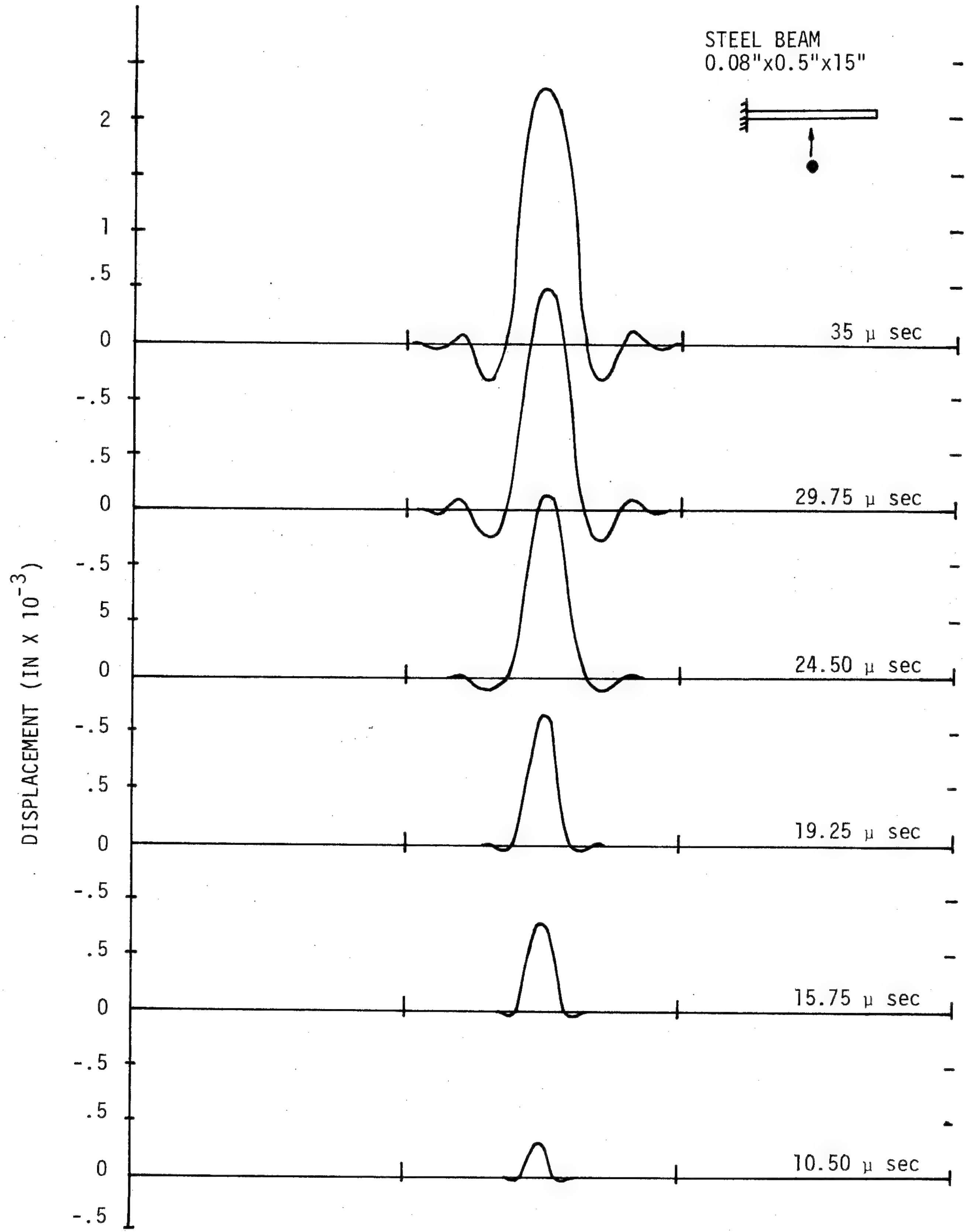

Fig. A-3 Displacement profiles at various times after impact of the steel beam.

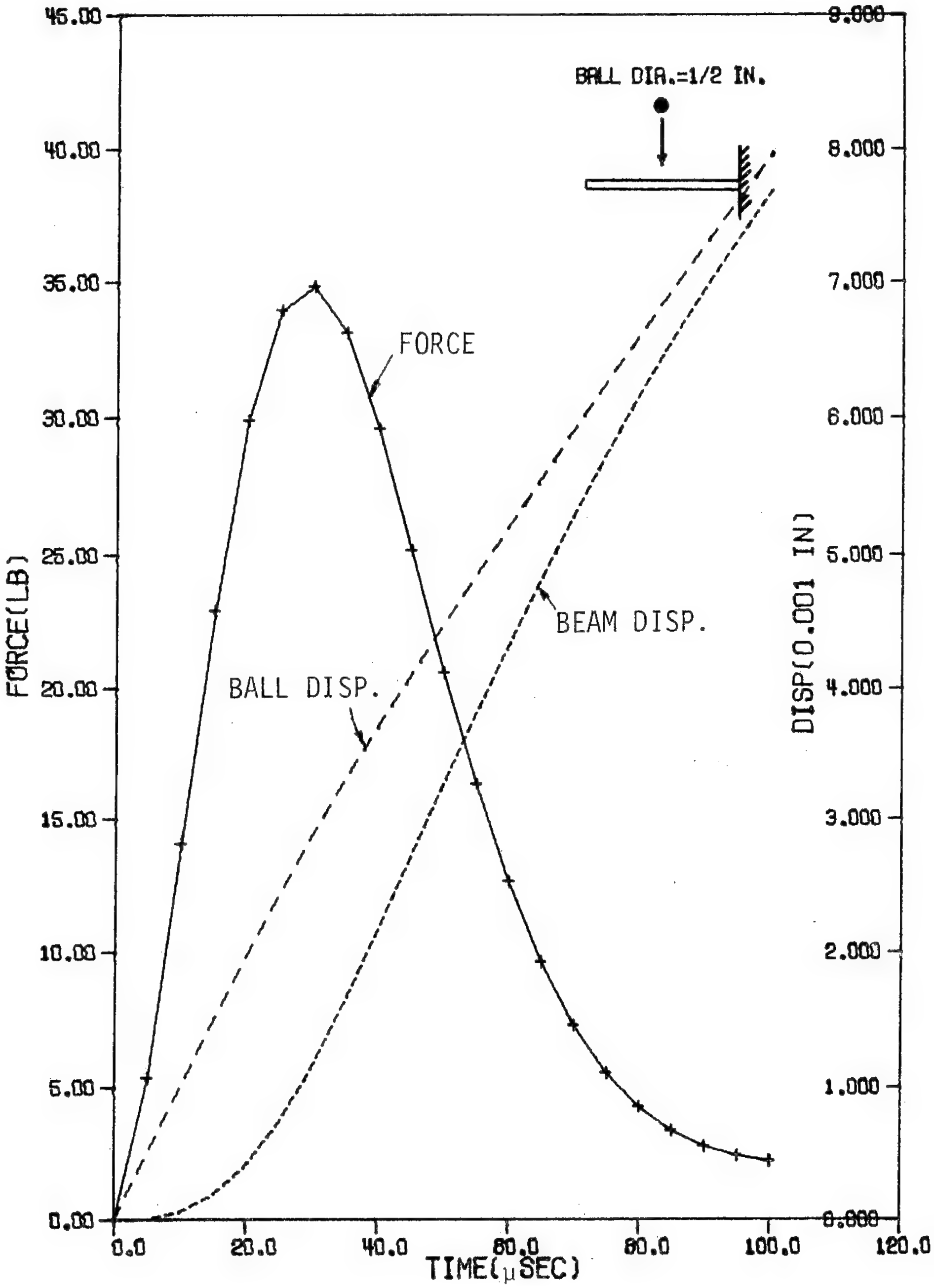

Fig. A-4 Response of a cantilever graphite/epoxy beam (0.5"W x 0.08"D x 15"L) subjected to impact of a steel ball at 100 in./sec.

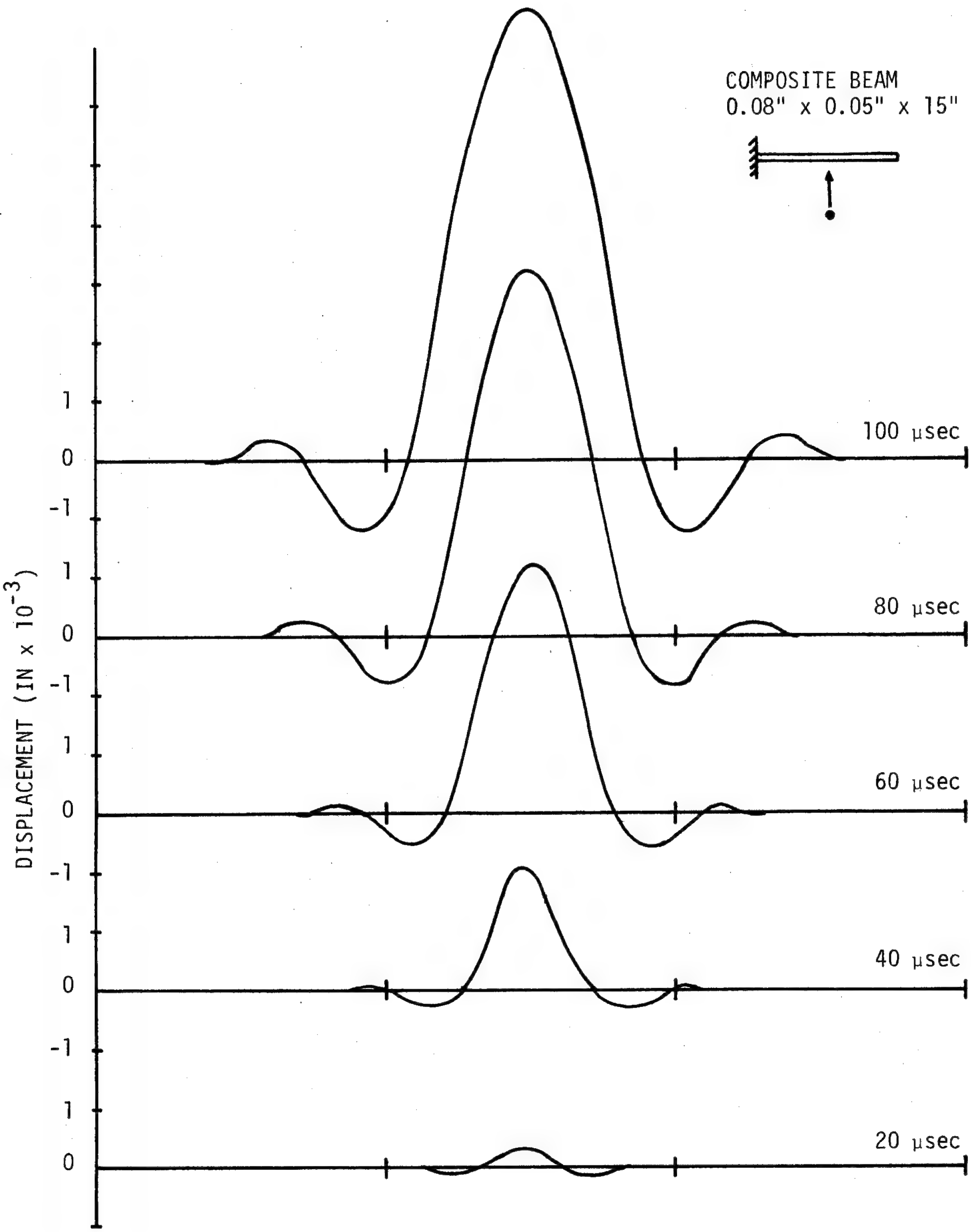

Fig. A-5 Displacement profiles at various times after impact of the composite beam.

Listing of Program

```
      PROGRAM MAIN (INPUT,OUTPUT,PLOT,TAPE5=INPUT,TAPE6=OUTPUT,TAPE11,TA
     1PE8)
C
C         CONTROL MAIN PROGRAM
C
      COMMON /CONTR/ TITLE(7),NP,NE,NB,NDF,NCN,NLD,NMAT,NSZF,LI,NT4,NDIN
     1,MATP,NPROB
      COMMON /TIME/ T,DT,DDT,TAU,KCON,KCNT
      COMMON /DISP/ Q1,Q2,Q3,Q10,Q20,Q30
      COMMON /DIMB/ TB,WB,PB,NQ,D11
      COMMON /SPHERE/ STF,R,CABU(10),QKONST(10)
      COMMON /PLASTIC/ DISPEM,NDISPEM,FORSPM,DISPM,QP
      COMMON CORD(100,2),NOP(200,4),IMAT(200),ORT(25,5),NBC(25),NFIX(25)
     1,R1(200),R2(200),R3(200),R10(200),R20(200),R30(200),FORS(200),SM(2
     200,15),SK(200,15),ISP(200,15),SMPEM(200,15),ESTIF(12,12),EMASS(12,
     312),NFIXK(25)
      COMMON /COMP/ QBR(3,3,25),ABD(6,6),TH(25),ZK(25),MLAYER
      COMMON /PLOT/ NN,TT(25),FF(25),W(25),V(25)
C
C        INITIALIZE TAPE NO.
C        AND NUMBER OF CORNER NODE MAX.
C
      NT4=11
      NCN=2
      NN=1
C
C       PROBLEM IDENTIFICATION
C
      CALL PLOTS
  101 READ (5,108) NPROB,(TITLE(I),I=1,7)
      IF (NPROB.EQ.0) GO TO 105
      DO 102 KG=1,200
         R10(KG)=0.
         R20(KG)=0.
         R30(KG)=0.
  102 R3(KG)=0.
C
C         READ INPUT GEOMETRY AND PROPERTIES
C
      CALL GDATA
      NDISPEM=0
      T=0.
      TAU=2.
      KCON=0
      DDT=DT*DT
C
C        LOOP ON NO OF PROBLEMS
C
      REWIND NT4
      NSZF=NP*NDF
      CALL FORMK
      CALL FORMM
      DO 103 LI=1,NLD
         KCNT=1
C
C        READ LOADS
C
         CALL LOAD
C
C        FORM THEN SOLVE SIMULTANEOUS EQUATIONS
C
         CALL HMTQ
         CALL SOLVE
         CALL INTEGTN
C
C        ITERATION 2
C
         KCNT=2
         CALL LOAD
         CALL HMTQ
```

```
      CALL SOLVE
      CALL INTEGTN
      T=T+DT
      IF (T.GT.100.E-6) GO TO 104
      IF (LI.EQ.10000) GO TO 104
103 CONTINUE
104 WRITE (6,106)
    WRITE (6,107) ((TT(I),FF(I),W(I),V(I)),I=1,NN)
    WRITE (8,107) ((TT(I),FF(I),W(I),V(I)),I=1,NN)
    CALL FACTOR (0.8)
    CALL PLOT (0.0,2.0,3)
    CALL SCALE (TT,6.0,21,1)
    CALL SCALE (FF,9.0,21,1)
    CALL SCALES (9.0,W,21,1,V,21,1)
C
C   W(22)=V(22)=TT(22)=FF(22)=0.0
C   TT(23)=20.
C   FF(23)=20.
C   W(23)=V(23)=0.001
C
    CALL AXIS (0.0,0.0,10HTIME( SEC),-10,6.0,0.0,TT(22),TT(23),0)
    CALL AXIS (0.0,0.0,9HFORCE(LB),9,9.0,90.0,FF(22),FF(23),-1)
    CALL AXIS (6.0,0.0,14HDISP(0.001 IN),14,9.0,90.0,W(22),W(23),-1)
    CALL LINE (TT,FF,21,1,1,3)
    CALL DSHLINE (TT,W,21,0.1,0.1,1)
    CALL DSHLINE (TT,V,21,0.05,0.05,1)
    CALL PLOT (6.0,9.0,3)
    CALL PLOT (0.0,9.0,2)
    CALL SYMBOL (1.0,9.3,0.1,TITLE,0.0,70)
    CALL SYMBOL (3.5,8.5,0.1,17HBALL DIA.=1/2 IN.,0.0,17)
    GO TO 101
105 CALL PLOT (0,0,999)
    STOP
C
106 FORMAT (1H1,4X,10HTIME(MSEC),6X,9HFORCE(LB),2X,13HBALL DISP(IN),2X
   1,13HBEAM DISP(IN))
107 FORMAT (4E15.3)
108 FORMAT (I2,7A10)
C
    END
    SUBROUTINE GDATA
    COMMON /CONTR/ TITLE(7),NP,NE,NB,NDF,NCN,NLD,NMAT,NSZF,LI,NT4,NDIN
   1,MATP,NPROB
    COMMON /TIME/ T,DT,DDT,TAU,KCON,KCNT
    COMMON /DISP/ Q1,Q2,Q3,Q10,Q20,Q30
    COMMON /DIMB/ TB,WB,PB,NQ,D11
    COMMON /SPHERE/ STF,R,CABV(10),QKONST(10)
    COMMON /PLASTIC/ DISPEM,NDISPEM,FORSPM,DISPM,QP
    COMMON CORD(100,2),NOP(200,4),IMAT(200),ORT(25,5),NBC(25),NFIX(25)
   1,R1(200),R2(200),R3(200),R10(200),R20(200),R30(200),FORS(200),SM(2
   200,15),SK(200,15),ISP(200,15),SMPEM(200,15),ESTIF(12,12),EMASS(12,
   312),NFIXK(25)
    COMMON /COMP/ QBR(3,3,25),ABD(6,6),TH(25),ZK(25),MLAYER
    COMMON /PLOT/ NN,TT(25),FF(25),W(25),V(25)
C
C      READ AND PRINT TITLE AND CONTROL
C
    WRITE (6,116) NPROB,(TITLE(I),I=1,7)
    WRITE (8,116) NPROB,(TITLE(I),I=1,7)
    READ (5,106) NP,NE,NB,NTM,NMAT,NDIN,MATP,NDC,I1
    NDF=3
    NLD=NDIN*NTM
    FLD=FLOAT(NLD)/10.
    FDIN=FLOAT(NDIN)/10.
    READ (5,113) TB,WB,R,NQ,Q2,DT
    WRITE (6,107) NP,NE,NB,NLD,NDF,NMAT
    NLD=NLD+1
C
C     READ AND PRINT MATERIAL DATA
C     SPHERE DATA: L=NMAT (LAST MAT. CARD)
```

```
C
      READ (5,112) (L,(ORT(L,I),I=1,5),N=1,NMAT)
      PB=ORT(NMAT,5)
      WRITE (6,122) TB,WB,PB,R,NQ,Q2,DT
      NQ=(NQ-1)*3+1
      WRITE (6,121)
      WRITE (6,115) (N,(ORT(N,I),I=1,5),N=1,NMAT)
C
C      READ INDENTATION DATA
C
      READ (5,111) STF,DISPEM,QP
      WRITE (6,123) DISPEM
      IF (DISPEM.NE.0.0) WRITE (6,124) QP
C
C         READ NODAL POINT DATA
C                  AND
C         READ ELEMENT DATA
C
      DO 102 I=1,NDC
         READ (5,114) ND1,ND2,X1,X2,IMT
         EL=(X2-X1)/FLOAT(ND2-ND1)
         CORD(ND1,1)=X1
         CORD(ND2,1)=X2
         CORD(ND2,2)=0.0
         CORD(ND1,2)=CORD(ND2,2)
         NDD=ND2-1
         DO 101 J=ND1,NDD
            CORD(J+1,1)=CORD(J,1)+EL
            CORD(J+1,2)=0.0
            NOP(J,1)=J
            NOP(J,2)=J+1
            NOP(J,4)=0
            NOP(J,3)=NOP(J,4)
            IMAT(J)=IMT
  101    CONTINUE
  102 CONTINUE
C
C         READ BOUNDARY DATA
C
      READ (5,110) (NBC(I),NFIX(I),I=1,NB)
      IF (MATP.EQ.1) CALL CMPD
C
C          ISOTROPIC        MATP=0.0
C          COMPOSITE        MATP=1.0
C
      IF (I1.NE.0) GO TO 103
C
C         PRINT INPUT DATA
C
      WRITE (6,117)
      WRITE (6,108) (N,(CORD(N,M),M=1,2),N=1,NP)
      WRITE (6,118)
      WRITE (6,109) (N,(NOP(N,M),M=1,4),IMAT(N),N=1,NE)
      WRITE (6,119)
      WRITE (6,110) (NBC(I),NFIX(I),I=1,NB)
      WRITE (6,120) FDIN,FLD
  103 CONTINUE
      DO 104 IJ=1,200
         R10(IJ)=0.
         R20(IJ)=0.
         R30(IJ)=0.
  104 FORS(IJ)=0.
      DO 105 IJ=1,25
  105 NFIXK(IJ)=NFIX(IJ)
      RETURN
C
  106 FORMAT (9I5)
  107 FORMAT (13H0NODAL POINTS,9X,I5/1X,8HELEMENTS,13X,I5/1X,19HBOUNDARY
     1 CONDITIONS,2X,I5/1X,12HOUTPUT LIMIT,10X,I5/1X,18HDEGREES OF FREED
     2OM,3X,I5/1X,9HMATERIALS,12X,I5)
```

```
  108 FORMAT (I10,2F10.3)
  109 FORMAT (6I5)
  110 FORMAT (2I5)
  111 FORMAT (E10.3,2F10.0)
  112 FORMAT (I5,5F10.0)
  113 FORMAT (3F10.0/I5,2F10.0)
  114 FORMAT (2I5,2F10.0,I5)
  115 FORMAT (I5,7X,3(F10.1,4X),F5.3,7X,F8.6//)
  116 FORMAT (1H1,I2,7A10)
  117 FORMAT (14H0 NODAL POINTS/17X,1HX,10X,1HY)
  118 FORMAT (10H0 ELEMENTS/9X,1HI,4X,1HJ,4X,1HK,8X,3HMAT)
  119 FORMAT (21H0 BOUNDARY CONDITIONS)
  120 FORMAT (16H0PRINTING SCHEME/5X,22H1. REPORT OUTPUT EVERY,F6.2,2X,4
     1HMSEC/5X,22H2. TERMINATE OUTPUT AT,F6.2,2X,4HMSEC)
  121 FORMAT (1H0,20H MATERIAL PROPERTIES/1X,8HMAT. NO.,7X,2HE1,12X,2HE2
     1,11X,3HG12,10X,3HV12,10X,3HRHO/)
  122 FORMAT (15H0BEAM THICKNESS,11X,F6.3/1X,10HBEAM WIDTH,15X,F6.3/1X,1
     14HSPHERE DENSITY,12X,F8.6/1X,13HSPHERE RADIUS,12X,F6.3//1X,11HIMPA
     2CT NODE,14X,I2/1X,15HIMPACT VELOCITY,10X,F6.1/1X, 39HINTEGRATION T
     3IME INCREMENT( X E-06 SEC),E10.3)
  123 FORMAT (//,1X, 21HPERMANENT DEFORMATION,9X,F8.6)
  124 FORMAT (/,1X, 15HUNLOADING POWER,15X,F6.3)
C
      END
      SUBROUTINE ESTIFM (N)
      REAL IB,LB
      COMMON /CONTR/ TITLE(7),NP,NE,NB,NDF,NCN,NLD,NMAT,NSZF,LI,NT4,NDIN
     1,MATP,NPROB
      COMMON /TIME/ T,DT,DDT,TAU,KCON,KCNT
      COMMON /DISP/ Q1,Q2,Q3,Q10,Q20,Q30
      COMMON /DIMB/ TB,WB,PB,NQ,D11
      COMMON CORD(100,2),NOP(200,4),IMAT(200),ORT(25,5),NBC(25),NFIX(25)
     1,R1(200),R2(200),R3(200),R10(200),R20(200),R30(200),FORS(200),SM(2
     200,15),SK(200,15),ISP(200,15),SMPEM(200,15),ESTIF(12,12),EMASS(12,
     312),NFIXK(25)
      COMMON /COMP/ QBR(3,3,25),ABD(6,6),TH(25),ZK(25),MLAYER
      IB=WB*TB**3/12.
      LB=CORD(N+1,1)-CORD(N,1)
      SQLB=LB*LB
      TPLB=LB*LB*LB
      IMN=IMAT(N)
      PARA1=ORT(IMN,1)*IB/70.
      IF (MATP.EQ.1) PARA1=ABD(4,4)/70.
      ESTIF(1,1)=1200./TPLB*PARA1
      ESTIF(1,2)=600./SQLB*PARA1
      ESTIF(1,3)=30./LB*PARA1
      ESTIF(1,4)=-1200./TPLB*PARA1
      ESTIF(1,5)=600./SQLB*PARA1
      ESTIF(1,6)=-30./LB*PARA1
      ESTIF(2,1)=ESTIF(1,2)
      ESTIF(2,2)=384./LB*PARA1
      ESTIF(2,3)=22.*PARA1
      ESTIF(2,4)=-600./SQLB*PARA1
      ESTIF(2,5)=216./LB*PARA1
      ESTIF(2,6)=-8.*PARA1
      ESTIF(3,1)=ESTIF(1,3)
      ESTIF(3,2)=ESTIF(2,3)
      ESTIF(3,3)=6.*LB*PARA1
      ESTIF(3,4)=-30./LB*PARA1
      ESTIF(3,5)=8.*PARA1
      ESTIF(3,6)=LB*PARA1
      ESTIF(4,1)=ESTIF(1,4)
      ESTIF(4,2)=ESTIF(2,4)
      ESTIF(4,3)=ESTIF(3,4)
      ESTIF(4,4)=1200./TPLB*PARA1
      ESTIF(4,5)=-600./SQLB*PARA1
      ESTIF(4,6)=30./LB*PARA1
      ESTIF(5,1)=ESTIF(1,5)
      ESTIF(5,2)=ESTIF(2,5)
      ESTIF(5,3)=ESTIF(3,5)
```

```
      ESTIF(5,4)=ESTIF(4,5)
      ESTIF(5,5)=384./LB*PARA1
      ESTIF(5,6)=-22.*PARA1
      ESTIF(6,1)=ESTIF(1,6)
      ESTIF(6,2)=ESTIF(2,6)
      ESTIF(6,3)=ESTIF(3,6)
      ESTIF(6,4)=ESTIF(4,6)
      ESTIF(6,5)=ESTIF(5,6)
      ESTIF(6,6)=6.*LB*PARA1
      IF (N.NE.1) GO TO 101
      WRITE (6,103)
      WRITE (6,102) ((ESTIF(I,J),J=1,6),I=1,6)
  101 CONTINUE
      RETURN
C
  102 FORMAT (1X,6E11.3)
  103 FORMAT (1H0,38H TYPICAL STIFNESS MATRIX OF AN ELEMENT)
C
      END
      SUBROUTINE EMASSM (N)
      REAL LB
      COMMON /CONTR/ TITLE(7),NP,NE,NB,NDF,NCN,NLD,NMAT,NSZF,LI,NT4,NDIN
     1,MATP,NPROB
      COMMON /TIME/ T,DT,DDT,TAU,KCON,KCNT
      COMMON /DISP/ Q1,Q2,Q3,Q10,Q20,Q30
      COMMON /DIMB/ TB,WB,PB,NQ,D11
      COMMON CORD(100,2),NOP(200,4),IMAT(200),ORT(25,5),NBC(25),NFIX(25)
     1,R1(200),R2(200),R3(200),R10(200),R20(200),R30(200),FORS(200),SM(2
     200,15),SK(200,15),ISP(200,15),SMPEM(200,15),ESTIF(12,12),EMASS(12,
     312),NFIXK(25)
      COMMON /COMP/ QBR(3,3,25),ABD(6,6),TH(25),ZK(25),MLAYER
      LB=CORD(N+1,1)-CORD(N,1)
      AB=TB*WB
      SQLB=LB*LB
      TPLB=LB*LB*LB
      QDLB=LB*LB*LB*LB
      IMN=IMAT(N)
      PARA2=ORT(IMN,5)*AB*LB/55440.
      EMASS(1,1)=21720.*PARA2
      EMASS(1,2)=3732.*LB*PARA2
      EMASS(1,3)=281.*SQLB*PARA2
      EMASS(1,4)=6000.*PARA2
      EMASS(1,5)=-1812.*LB*PARA2
      EMASS(1,6)=181.*SQLB*PARA2
      EMASS(2,1)=EMASS(1,2)
      EMASS(2,2)=832.*SQLB*PARA2
      EMASS(2,3)=69.*TPLB*PARA2
      EMASS(2,4)=1812.*LB*PARA2
      EMASS(2,5)=-532.*SQLB*PARA2
      EMASS(2,6)=52.*TPLB*PARA2
      EMASS(3,1)=EMASS(1,3)
      EMASS(3,2)=EMASS(2,3)
      EMASS(3,3)=6.*QDLB*PARA2
      EMASS(3,4)=181.*SQLB*PARA2
      EMASS(3,5)=-52.*TPLB*PARA2
      EMASS(3,6)=5.*QDLB*PARA2
      EMASS(4,1)=EMASS(1,4)
      EMASS(4,2)=EMASS(2,4)
      EMASS(4,3)=EMASS(3,4)
      EMASS(4,4)=21720.*PARA2
      EMASS(4,5)=-3732.*LB*PARA2
      EMASS(4,6)=281.*SQLB*PARA2
      EMASS(5,1)=EMASS(1,5)
      EMASS(5,2)=EMASS(2,5)
      EMASS(5,3)=EMASS(3,5)
      EMASS(5,4)=EMASS(4,5)
      EMASS(5,5)=832.*SQLB*PARA2
      EMASS(5,6)=-69.*TPLB*PARA2
      EMASS(6,1)=EMASS(1,6)
      EMASS(6,2)=EMASS(2,6)
```

```
      EMASS(6,3)=EMASS(3,6)
      EMASS(6,4)=EMASS(4,6)
      EMASS(6,5)=EMASS(5,6)
      EMASS(6,6)=6.*QDLB*PARA2
      IF (N.NE.1) GO TO 101
      WRITE (6,103)
      WRITE (6,102) ((EMASS(I,J),J=1,6),I=1,6)
  101 CONTINUE
      RETURN
C
  102 FORMAT (1X,6E11.3)
  103 FORMAT (1H0,34H TYPICAL MASS MATRIX OF AN ELEMENT)
C
      END
      SUBROUTINE FORMM
C
C         FORMS MASS MATRIX
C         IN UPPER TRIANGULAR FORM
C
      COMMON /CONTR/ TITLE(7),NP,NE,NB,NDF,NCN,NLD,NMAT,NSZF,LI,NT4,NDIN
     1,MATP,NPROB
      COMMON /TIME/ T,DT,DDT,TAU,KCON,KCNT
      COMMON /DISP/ Q1,Q2,Q3,Q10,Q20,Q30
      COMMON /DIMB/ TB,WB,PB,NQ,D11
      COMMON CORD(100,2),NOP(200,4),IMAT(200),ORT(25,5),NBC(25),NFIX(25)
     1,R1(200),R2(200),R3(200),R10(200),R20(200),R30(200),FORS(200),SM(2
     200,15),SK(200,15),ISP(200,15),SMPEM(200,15),ESTIF(12,12),EMASS(12,
     312),NFIXK(25)
      COMMON /COMP/ QBR(3,3,25),ABD(6,6),TH(25),ZK(25),MLAYER
C
C         SET BANDMAX AND NO. OF EQUATIONS
C
      NBAND=9
C
C         ZERO MASS MATRIX
C
      DO 101 N=1,NSZF
      DO 101 M=1,NBAND
  101 SM(N,M)=0.
C
C         SCAN ELEMENTS
C
      DO 106 N=1,NE
         CALL EMASSM (N)
C
C         RETURNS EMASS AS MASS MATRIX
C
C         STORE EMASS IN SM
C
C         FIRST ROWS
C
         DO 105 JJ=1,NCN
            NROWB=(NOP(N,JJ)-1)*NDF
         DO 105 J=1,NDF
            NROWB=NROWB+1
            I=(JJ-1)*NDF+J
C
C         THEN COLUMNS
C
            DO 104 KK=1,NCN
               NCOLB=(NOP(N,KK)-1)*NDF
               DO 103 K=1,NDF
                  L=(KK-1)*NDF+K
                  NCOL=NCOLB+K+1-NROWB
C
C         SKIP STORING IF BELOW BAND
C
                  IF (NCOL) 103,103,102
  102             SM(NROWB,NCOL)=SM(NROWB,NCOL)+EMASS(I,L)
  103          CONTINUE
```

```
 104        CONTINUE
 105     CONTINUE
 106 CONTINUE
C
C        INSERT BOUNDARY CONDITIONS
C
     DO 112 N=1,NB
        NX=10**(NDF-1)
        I=NBC(N)
        NROWB=(I-1)*NDF
C
C        EXAMINE EACH DEGREE OF FREEDOM
C
        DO 111 M=1,NDF
           NROWB=NROWB+1
           ICON=NFIX(N)/NX
           IF (ICON) 110,110,107
 107       SM(NROWB,1)=1.
           DO 109 J=2,NBAND
              SM(NROWB,J)=0.
              NR=NROWB+1-J
              IF (NR) 109,109,108
 108          SM(NR,J)=0.
 109       CONTINUE
           NFIX(N)=NFIX(N)-NX*ICON
 110       NX=NX/10
 111    CONTINUE
 112 CONTINUE
     DO 115 N=1,NSZF
        K=0
        DO 114 M=1,NBAND
           MP=M-K
           IF (ISP(N,M).LT.ISP(N,1)) GO TO 113
           SM(N,MP)=SM(N,MP)+(DDT/6.)*SK(N,M)
           GO TO 114
 113       K=K+1
 114    CONTINUE
 115 CONTINUE
     DO 116 I=1,NSZF
     DO 116 J=1,NBAND
 116 SMPEM(I,J)=SM(I,J)
C
C    WRITE(6,1) ((SM(I,J),J=1,NBAND),I=1,NSZF)
C    1 FORMAT(2X,9E10.3)
C
     RETURN
C
     END
     SUBROUTINE FORMK
     COMMON /CONTR/ TITLE(7),NP,NE,NB,NDF,NCN,NLD,NMAT,NSZF,LI,NT4,NDIN
    1,MATP,NPROB
     COMMON /TIME/ T,DT,DDT,TAU,KCON,KCNT
     COMMON /DISP/ Q1,Q2,Q3,Q10,Q20,Q30
     COMMON /DIMB/ TB,WB,PB,NQ,D11
     COMMON CORD(100,2),NOP(200,4),IMAT(200),ORT(25,5),NBC(25),NFIX(25)
    1,R1(200),R2(200),R3(200),R10(200),R20(200),R30(200),FORS(200),SM(2
    200,15),SK(200,15),ISP(200,15),SMPEM(200,15),ESTIF(12,12),EMASS(12,
    312),NFIXK(25)
     COMMON /COMP/ QBR(3,3,25),ABD(6,6),TH(25),ZK(25),MLAYER
C
C        SET MAX. NO. OF TERMS
C
     NMAX=9
     NOFF=9
C
C        ZERO ARRAYS
C
     DO 103 N=1,NSZF
        DO 101 M=1,NMAX
 101    SK(N,M)=0.
```

```
          DO 102 M=2,NOFF
  102     ISP(N,M)=0
  103 ISP(N,1)=N
C
C         SCAN ELEMENTS
C
      DO 110 N=1,NE
          CALL ESTIFM (N)
C
C         RETURNS ESTIF AS STIFFNESS MATRIX
C
C         STORE ESTIF IN SK WITH A TERM IN ISP AS A POINTER
C
C
C         FIRST THE ROWS
C
          I=0
          DO 109 JJ=1,NCN
             NROWB=(NOP(N,JJ)-1)*NDF
          DO 109 J=1,NDF
             NROWB=NROWB+1
             I=I+1
C
C         THEN COLUMNS OF ESTIF
C
             II=0
             DO 108 KK=1,NCN
                NCOLB=(NOP(N,KK)-1)*NDF
             DO 108 K=1,NDF
                NCOLB=NCOLB+1
                II=II+1
C
C         SEARCH ISP FOR COLUMN NO.
C
                DO 105 M=1,NOFF
                   IF (ISP(NROWB,M)-NCOLB) 104,107,104
  104              IF (ISP(NROWB,M)) 106,106,105
  105           CONTINUE
C
C         FOUND A BLANK NOW STORE NCOLB
C
  106           ISP(NROWB,M)=NCOLB
C
C         NOW STORE ESTIF
C
  107           SK(NROWB,M)=ESTIF(I,II)+SK(NROWB,M)
C
C         END LOOP ON COLUMNS
C
  108        CONTINUE
C
C         END LOOP ON ROWS
C
  109     CONTINUE
C
C         END LOOP ON ELEMENTS
C
  110 CONTINUE
C
C         INSERT BOUNDARY CONDITIONS
C
      DO 114 N=1,NB
          NX=10**(NDF-1)
          I=NBC(N)
          NROWB=(I-1)*NDF
C
C         EXAMINE EACH DEGREE OF FREEDOM
C
          DO 113 M=1,NDF
             NROWB=NROWB+1
```

```
          ICON=NFIXK(N)/NX
          IF (ICON) 112,112,111
C
C       STORE ZERO ON DIAGONAL
C
 111      SK(NROWB,1)=0.0
          NFIXK(N)=NFIXK(N)-NX*ICON
 112      NX=NX/10
 113    CONTINUE
 114  CONTINUE
      RETURN
C
      END
      SUBROUTINE LOAD
      COMMON /CONTR/ TITLE(7),NP,NE,NB,NDF,NCN,NLD,NMAT,NSZF,LI,NT4,NDIN
     1,MATP,NPROB
      COMMON /TIME/ T,DT,DDT,TAU,KCON,KCNT
      COMMON /DISP/ Q1,Q2,Q3,Q10,Q20,Q30
      COMMON /DIMB/ TB,WB,PB,NQ,D11
      COMMON /SPHERE/ STF,R,CABV(10),QKONST(10)
      COMMON /PLASTIC/ DISPEM,NDISPEM,FORSPM,DISPM,QP
      COMMON CORD(100,2),NOP(200,4),IMAT(200),ORT(25,5),NBC(25),NFIX(25)
     1,R1(200),R2(200),R3(200),R10(200),R20(200),R30(200),FORS(200),SM(2
     200,15),SK(200,15),ISP(200,15),SMPEM(200,15),ESTIF(12,12),EMASS(12,
     312),NFIXK(25)
      COMMON /COMP/ QBR(3,3,25),ABD(6,6),TH(25),ZK(25),MLAYER
      IF (STF.NE.0.0) GO TO 101
      STFI=(4./3.)*SQRT(R)/((1.-ORT(NMAT,4)**2)/ORT(NMAT,1)+(1.-ORT(1,4)
     1**2)/ORT(1,1))
      STFA=(4./3.)*SQRT(R)/((1.-ORT(NMAT,4)**2)/ORT(NMAT,1)+1./ORT(1,2))
      STF=STFI
      IF (MATP.EQ.1) STF=STFA
 101  PAI=4.*ATAN(1.)
      BALLM=(4./3.)*PAI*(R**3)*PB
C
C        SIMPLY SUPPORTED SYMMETRY  CST1=0.5
C        CLAMPED CANTILEVER CST1=1.
C
C
      CST1=1.0
C
C     IF(NBC(1) .EQ. 1) CST1=0.5
C
C        Q1=ACCEL. OF THE MASS
C        Q2=VELO. OF THE MASS
C        Q3=DISP. OF THE MASS
C
      IF (LI.GT.1.AND.KCNT.EQ.1) GO TO 102
      IF (LI.GT.1.AND.KCNT.EQ.2) GO TO 103
      Q1=0.
      Q3=0.
      GO TO 112
 102  Q10=Q1
      Q20=Q2
      Q30=Q3
      Q3=Q30+DT*Q20+0.5*DDT*Q10
      R3(NQ)=R30(NQ)+DT*R20(NQ)+0.5*DDT*R10(NQ)
      DIFDO=Q30-R30(NQ)
      DIFDISP=Q3-R3(NQ)
C
C     WRITE(6,400) DIFDO,DIFDISP
C
      IF (DIFDISP) 110,104,104
 103  Q3=Q30+DT*Q20+DDT*Q10/3.+DDT*Q1/6.
      DIFDO=Q30-R30(NQ)
      DIFDISP=Q3-R3(NQ)
C
C     WRITE(6,400) DIFDO,DIFDISP
C     400 FORMAT(/,5X,≠DIFDO=≠,E15.3,5X,≠DIFDISP=≠,E15.3)
C
```

```
      IF (DIFDISP.LT.0) GO TO 110
  104 IF (DISPEM.EQ.0.0) GO TO 105
      IF ((DIFDISP.LT.DIFDO).AND.(NDISPEM.EQ.0)) GO TO 107
      IF ((DIFDISP.LT.DIFDO).AND.(NDISPEM.GT.0)) GO TO 108
  105 DO 106 J=1,NSZF
  106 FORS(J)=0.
      FORS(NQ)=STF*(DIFDISP)**1.5*CST1
      Q1=-FORS(NQ)/BALLM/CST1
      IF (KCNT.EQ.1) GO TO 113
      IF (KCNT.EQ.2) GO TO 109
  107 NDISPEM=1
      FORSPM=FORS(NQ)
      DISPM=DIFDO
      WRITE (6,114) DISPEM,DISPM,DIFDISP,FORSPM
      IF ((DIFDISP.LT.DISPEM).OR.(DISPM.LE.DISPEM)) GO TO 111
  108 FORS(NQ)=FORSPM*((DIFDISP-DISPEM)/(DISPM-DISPEM))**QP*CST1
      Q1=-FORS(NQ)/BALLM/CST1
      IF (KCNT.EQ.1) GO TO 113
  109 Q2=Q20+0.5*DT*Q10+0.5*DT*Q1
      Q3=Q30+DT*Q20+DDT*Q10/3.+DDT*Q1/6.
      GO TO 113
  110 FORS(NQ)=0.
      Q1=0.
      GO TO 109
  111 LI=10000
      GO TO 113
  112 FORS(NQ)=0.
  113 RETURN
C
  114 FORMAT (///,5X,  7HDISPEM=,E10.3,5X,  6HDISPM=,E10.3,5X,  8HDIFDIS
     1P=,E10.3,5X,  7HFORSPM=,E10.3)
C
      END
      SUBROUTINE HMTQ
C
C        SUBROUTINE FOR FINDING (F)-(K)(U)
C
      COMMON /CONTR/ TITLE(7),NP,NE,NB,NDF,NCN,NLD,NMAT,NSZF,LI,NT4,NDIN
     1,MATP,NPROB
      COMMON /TIME/ T,DT,DDT,TAU,KCON,KCNT
      COMMON /DISP/ Q1,Q2,Q3,Q10,Q20,Q30
      COMMON /DIMB/ TB,WB,PB,NQ,D11
      COMMON CORD(100,2),NOP(200,4),IMAT(200),ORT(25,5),NBC(25),NFIX(25)
     1,R1(200),R2(200),R3(200),R10(200),R20(200),R30(200),FORS(200),SM(2
     200,15),SK(200,15),ISP(200,15),SMPEM(200,15),ESTIF(12,12),EMASS(12,
     312),NFIXK(25)
      COMMON /COMP/ QBR(3,3,25),ABD(6,6),TH(25),ZK(25),MLAYER
      NT=9
      DO 101 IJ=1,NSZF
         R1(IJ)=0.
         R2(IJ)=0.
  101 R3(IJ)=0.
C
C
      DO 105 N=1,NSZF
         FX=FORS(N)
         DO 102 M=1,NT
            L=ISP(N,M)
  102    FX=FX-SK(N,M)*(R30(L)+DT*R20(L)+(DDT/3.)*R10(L))
         IF (SK(N,1)) 104,103,104
  103    FX=0.
  104    R1(N)=FX
  105 CONTINUE
      RETURN
C
      END
      SUBROUTINE SOLVE
C
C        SPECIFICATION STATEMENTS
C
```

```
      COMMON /CONTR/ TITLE(7),NP,NE,NB,NDF,NCN,NLD,NMAT,NSZF,LI,NT4,NDIN
     1,MATP,NPROB
      COMMON /TIME/ T,DT,DDT,TAU,KCON,KCNT
      COMMON /DISP/ Q1,Q2,Q3,Q10,Q20,Q30
      COMMON /DIMB/ TB,WB,PB,NQ,D11
      COMMON CORD(100,2),NOP(200,4),IMAT(200),ORT(25,5),NBC(25),NFIX(25)
     1,R1(200),R2(200),R3(200),R10(200),R20(200),R30(200),FORS(200),SM(2
     200,15),SK(200,15),ISP(200,15),SMPEM(200,15),ESTIF(12,12),EMASS(12,
     312),NFIXK(25)
      COMMON /COMP/ QBR(3,3,25),ABD(6,6),TH(25),ZK(25),MLAYER
      NBAND=9
      DO 101 I=1,NSZF
      DO 101 J=1,NBAND
  101 SM(I,J)=SMPEM(I,J)
C
C         REDUCE MATRIX
C
      DO 106 N=1,NSZF
         I=N
         DO 105 L=2,NBAND
            I=I+1
            IF (SM(N,L)) 102,105,102
  102       C=SM(N,L)/SM(N,1)
            J=0
            DO 104 K=L,NBAND
               J=J+1
               IF (SM(N,K)) 103,104,103
  103          SM(I,J)=SM(I,J)-C*SM(N,K)
  104       CONTINUE
            SM(N,L)=C
C
C         AND LOAD VECTOR
C         FOR EACH EQUATION
C
            R1(I)=R1(I)-C*R1(N)
  105    CONTINUE
  106 R1(N)=R1(N)/SM(N,1)
C
C         BACK-SUBSTITUTION
C
      N=NSZF
  107 N=N-1
      IF (N) 111,111,108
  108 L=N
      DO 110 K=2,NBAND
         L=L+1
         IF (SM(N,K)) 109,110,109
  109    R1(N)=R1(N)-SM(N,K)*R1(L)
  110 CONTINUE
      GO TO 107
  111 RETURN
C
      END
      SUBROUTINE INTEGTN
      COMMON /CONTR/ TITLE(7),NP,NE,NB,NDF,NCN,NLD,NMAT,NSZF,LI,NT4,NDIN
     1,MATP,NPROB
      COMMON /TIME/ T,DT,DDT,TAU,KCON,KCNT
      COMMON /DISP/ Q1,Q2,Q3,Q10,Q20,Q30
      COMMON /DIMB/ TB,WB,PB,NQ,D11
      COMMON CORD(100,2),NOP(200,4),IMAT(200),ORT(25,5),NBC(25),NFIX(25)
     1,R1(200),R2(200),R3(200),R10(200),R20(200),R30(200),FORS(200),SM(2
     200,15),SK(200,15),ISP(200,15),SMPEM(200,15),ESTIF(12,12),EMASS(12,
     312),NFIXK(25)
      COMMON /COMP/ QBR(3,3,25),ABD(6,6),TH(25),ZK(25),MLAYER
      COMMON /PLOT/ NN,TT(25),FF(25),W(25),V(25)
C
C         R1=ACCEL. OF BEAM
C         R2=VELO. OF BEAM
C         R3=DISPL. OF BEAM
C
```

```
      DO 101 IJ=1,NSZF
         R2(IJ)=R20(IJ)+0.5*DT*R10(IJ)+0.5*DT*R1(IJ)
         R3(IJ)=R30(IJ)+DT*R20(IJ)+(DDT/3.)*R10(IJ)+(DDT/6.)*R1(IJ)
  101 CONTINUE
      IF (KCNT.EQ.1) GO TO 107
      DO 102 IK=1,NP
         IK4=(IK-1)*3+1
         R30(IK)=R3(IK4)
  102 CONTINUE
      IF ((LI/10000).EQ.1) GO TO 103
C
C        PRINT CONTROL
C
      NTON=(LI-1)/NDIN
      IF (NTON.NE.KCON) GO TO 105
  103 CONTINUE
C
C        SIMPLY SUPPORTED BEAM CST2=2.
C        CANTILEVER CST2=1.
C
      CST2=1.
C
C     IF(NBC(1).EQ.1)CST2=2.
C
      F=CST2*FORS(NQ)
      APHA=Q3-R30(NQ)
      FF(NN)=F
      W(NN)=Q3*1000.
      V(NN)=R30(NQ)*1000.
      WRITE (6,108) (TITLE(I),I=1,7)
      T1=T*1.E6
      TT(NN)=T1
      NN=NN+1
      WRITE (6,109) T1,F,Q3,Q2,Q1,APHA
      DO 104 IK=1,NP
         IK3=IK*3
         STXX=R3(IK3)*TB/2.
         SIGX=ORT(1,1)*STXX
         STYY=-ORT(1,4)*STXX
         WRITE (6,110) IK,R30(IK),STXX,STYY,SIGX
  104 CONTINUE
      KCON=KCON+1
  105 CONTINUE
      DO 106 IJ=1,NSZF
         R10(IJ)=R1(IJ)
         R20(IJ)=R2(IJ)
  106 R30(IJ)=R3(IJ)
  107 RETURN
C
  108 FORMAT (1H1,7A10///)
  109 FORMAT (10X,18HTIME ELAPSED(MSEC),13X,F7.3/10X,9HFORCE(LB),21X,E11
     1.3/10X,21HMASS DISPLACEMENT(IN),9X,E11.3/10X,21HMASS VELOCITY(IN/S
     2EC),9X,E11.3/10X,20HMASS ACCEL.(IN/SEC2),10X,E11.3/10X,15HINDENTAT
     3ION(IN),15X,E11.3///10X,4HNODE,9X,4HDISP,13X,9HSTRAIN-XX,9X,9HSTRA
     4IN-YY,9X,9HSTRESS-XX/)
  110 FORMAT (9X,I3,7X,E12.3,7X,E12.3,7X,E12.3,7X,E12.3)
C
      END
      SUBROUTINE CMPD
      COMMON /CONTR/ TITLE(7),NP,NE,NB,NDF,NCN,NLD,NMAT,NSZF,LI,NT4,NDIN
     1,MATP,NPROB
      COMMON /TIME/ T,DT,DDT,TAU,KCON,KCNT
      COMMON /DISP/ Q1,Q2,Q3,Q10,Q20,Q30
      COMMON /DIMB/ TB,WB,PB,NQ,D11
      COMMON CORD(100,2),NOP(200,4),IMAT(200),ORT(25,5),NBC(25),NFIX(25)
     1,R1(200),R2(200),R3(200),R10(200),R20(200),R30(200),FORS(200),SM(2
     200,15),SK(200,15),ISP(200,15),SMPEM(200,15),ESTIF(12,12),EMASS(12,
     312),NFIXK(25)
      COMMON /COMP/ QBR(3,3,25),ABD(6,6),TH(25),ZK(25),MLAYER
      DIMENSION Q(3,3), TK(25)
```

```
      DO 102 J=1,3
      DO 102 K=1,3
         ABD(J+3,K+3)=0.0
         ABD(J+3,K)=ABD(J+3,K+3)
         ABD(J,K+3)=ABD(J+3,K)
         ABD(J,K)=ABD(J,K+3)
         DO 101 I=1,25
            QBR(J,K,I)=0.
  101    CONTINUE
  102 CONTINUE
      READ (5,108) MLAYER
      M=MLAYER
      READ (5,109) (L,TH(L),TK(L),I=1,M)
      TTK=0.0
      ZK(1)=TTK
      DO 103 I=1,M
         TTK=TTK+TK(I)
         ZK(I+1)=TK(I)+ZK(I)
  103 CONTINUE
      MM=M+1
      DO 104 I=1,MM
         ZK(I)=ZK(I)-TTK/2.
  104 CONTINUE
      DEL=4.*ATAN(1.)/180.
      DEN=1.-ORT(1,2)*ORT(1,4)**2/ORT(1,1)
      Q(1,1)=ORT(1,1)/DEN
      Q(2,2)=ORT(1,2)/DEN
      Q(2,1)=ORT(1,4)*Q(2,2)
      Q(1,2)=Q(2,1)
      Q(3,3)=ORT(1,3)
      Q(3,2)=0.0
      Q(3,1)=Q(3,2)
      Q(2,3)=Q(3,1)
      Q(1,3)=Q(2,3)
      DO 105 I=1,M
         ANGL=TH(I)*DEL
         C=COS(ANGL)
         W=SIN(ANGL)
         QBR(1,1,I)=Q(1,1)*C**4+2.*(Q(1,2)+2.*Q(3,3))*(C*W)**2+Q(2,2)*W*
     1   *4
         QBR(2,1,I)=(Q(1,1)+Q(2,2)-4.*Q(3,3))*(C*W)**2+Q(1,2)*(W**4+C**4
     1   )
         QBR(1,2,I)=QBR(2,1,I)
         QBR(2,2,I)=Q(1,1)*W**4+2.*(Q(1,2)+2.*Q(3,3))*(C*W)**2+Q(2,2)*C*
     1   *4
         QBR(3,1,I)=(Q(1,1)-Q(1,2)-2.*Q(3,3))*W*C**3+(Q(1,2)-Q(2,2)+(2.)
     1   *Q(3,3))*(W)*(C**3)
         QBR(1,3,I)=QBR(3,1,I)
         QBR(3,2,I)=(Q(1,1)-Q(1,2)-2.*Q(3,3))*W**3*C+(Q(1,2)-Q(2,2)+2.*Q
     1   (3,3))*W*C**3
         QBR(2,3,I)=QBR(3,2,I)
         QBR(3,3,I)=(Q(1,1)+Q(2,2)-2.*Q(1,2)-2.*Q(3,3))*(W*C)**2+Q(3,3)*
     1   (W**4+C**4)
C
C     WRITE(6,500) I,TH(I),TK(I)
C     WRITE(6,510)
C     WRITE(6,520) ((QBR(J,K,I),K=1,3),J=1,3)
C
  105 CONTINUE
      DO 107 J=1,3
      DO 107 K=1,3
         DO 106 I=1,M
            ABD(J,K)=ABD(J,K)+QBR(J,K,I)*(ZK(I+1)-ZK(I))
            ABD(J,K+3)=ABD(J+3,K)+QBR(J,K,I)*(ZK(I+1)**2-ZK(I)**2)/2.
            ABD(J+3,K)=ABD(J,K+3)
            ABD(J+3,K+3)=ABD(J+3,K+3)+QBR(J,K,I)*(ZK(I+1)**3-ZK(I)**3)/3
     1      .
  106    CONTINUE
  107 CONTINUE
      WRITE (6,110)
```

```
      WRITE (6,111) ((ABD(I,J),J=1,6),I=1,6)
C
C     500 FORMAT(2X,*LAYER=*,I2,5X,*ANGLE=*,F5.2,5X,*THICKNESS=*,F7.3)
C     510 FORMAT(2X,*QBAR-MATRIX*)
C     520 FORMAT(5X,3E12.3/)
C
      RETURN
C
  108 FORMAT (I5)
  109 FORMAT (I5,F5.0,F10.0)
  110 FORMAT (///,1X,10HABD MATRIX)
  111 FORMAT (1X,6E11.3)
C
      END
```

APPENDIX B

A COMPUTER PROGRAM FOR ESTIMATING THE CONTACT FORCE HISTORY BY USING THE EQUIVALENT MASS MODEL

This program has been written for simply-supported beams only. Cantilever beams and simply-supported plates will be added in the near future.

This program will be a subprogram in a large finite element program capable of analyzing impact responses of beams and plates. This subprogram will be used to provide an estimate of the contact time so that one may select a proper time increment for the finite difference used in the program. For this reason, the input cards for this subprogram were written to be identical to that for the program presented in Appendix A.

Listing of Program

```
      PROGRAM MAIN (INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
      COMMON /CONTR/ TITLE(10),NP,NE,NB,NDF,NCN,NLD,NMAT,NSZF,LI,NT4,ND+
     1N,MATP,NPROB
      COMMON /TIME/ T,DT,DDT,TAU,KCON,KCNT
      COMMON /DISP/ Q1,Q2,Q3,Q10,Q20,Q30
      COMMON /DIMB/ TB,WB,PB,NQ,D11
      COMMON /SPHERE/ STF,R,CABV(10),QKONST(10)
      COMMON CORD(100,2),NOP(200,4),IMAT(200),ORT(25,5),NBC(25),NFIX(25)
     1,R1(200),R2(200),R3(200),R10(200),R20(200),R30(200),FORS(200),SM(2
     200,15),SK(200,15),ISP(200,15),SMPEM(200,15),ESTIF(12,12),EMASS(12,
     312),NFIXK(25)
      COMMON /COMP/ QBR(3,3,25),ABD(6,6),TH(25),ZK(25),MLAYER
      COMMON X1,X2,ND1,ND2
      REAL IB
C
C        READ AND PRINT TITLE AND CONTROL
C
  101 READ (5,117) NPROB,(TITLE(I),I=1,10)
      IF (NPROB.EQ.0) GO TO 107
      WRITE (6,118) NPROB,(TITLE(I),I=1,10)
      READ (5,108) NP,NE,NB,NTM,NMAT,NDIN,MATP,NDC,I1
      NDF=3
      NLD=NDIN*NTM
      FLD=FLOAT(NLD)/10.
      FDIN=FLOAT(NDIN)/10.
      READ (5,114) TB,WB,R,NQ,Q2,DT
      WRITE (6,109) NP,NE,NB,NLD,NDF,NMAT
      NLD=NLD+1
C
C      READ AND PRINT MATERIAL DATA
C      SPHERE DATA: L=NMAT (LAST MAT. CARD)
C
      READ (5,113) (L,(ORT(L,I),I=1,5),N=1,NMAT)
      PB=ORT(NMAT,5)
      WRITE (6,123) TB,WB,PB,R,NQ,Q2,DT
      WRITE (6,122)
      WRITE (6,116) (N,(ORT(N,I),I=1,5),N=1,NMAT)
C
C      READ INDENTATION CARD
C
      READ (5,112) STF
C
C        READ NODAL POINT DATA
C                 AND
C        READ ELEMENT DATA
C
      DO 103 I=1,NDC
         READ (5,115) ND1,ND2,X1,X2,IMT
         EL=(X2-X1)/FLOAT(ND2-ND1)
         CORD(ND1,1)=X1
         CORD(ND2,1)=X2
         CORD(ND2,2)=0.0
         CORD(ND1,2)=CORD(ND2,2)
         NDD=ND2-1
         DO 102 J=ND1,NDD
            CORD(J+1,1)=CORD(J,1)+EL
            CORD(J+1,2)=0.0
            NOP(J,1)=J
            NOP(J,2)=J+1
            NOP(J,4)=0
            NOP(J,3)=NOP(J,4)
            IMAT(J)=IMT
  102    CONTINUE
  103 CONTINUE
C
C        READ BOUNDARY DATA
C
      READ (5,111) (NBC(I),NFIX(I),I=1,NB)
      IF (MATP.EQ.1) CALL CMPD
C
```

```
C          ISOTROPIC        MATP=0.0
C          COMPOSITE        MATP=1.0
C
      IF (I1.NE.0) GO TO 104
C
C        PRINT INPUT DATA
C
      WRITE (6,119)
      WRITE (6,110) (N,(CORD(N,M),M=1,2),N=1,NP)
C
C     WRITE(6,103)
C     WRITE(6,3)(N,(NOP(N,M),M=1,4),IMAT(N),N=1,NE)
C
      WRITE (6,120)
      WRITE (6,111) (NBC(I),NFIX(I),I=1,NB)
      WRITE (6,121) FDIN,FLD
  104 CONTINUE
      DO 105 IJ=1,200
         R10(IJ)=0.
         R20(IJ)=0.
         R30(IJ)=0.
  105 FORS(IJ)=0.
      DO 106 IJ=1,25
  106 NFIXK(IJ)=NFIX(IJ)
      CALL TMX
C
C      3 FORMAT(6I5)
C
C      103 FORMAT(10H0 ELEMENTS/9X,1HI,4X,1HJ,4X,1HK,8X,3HMAT)
C
      GO TO 101
  107 STOP
C
  108 FORMAT (9I5)
  109 FORMAT (13H0NODAL POINTS,9X,I5/1X,8HELEMENTS,13X,I5/1X,19HBOUNDARY
     1 CONDITIONS,2X,I5/1X,12HOUTPUT LIMIT,10X,I5/1X,18HDEGREES OF FREED
     2OM,3X,I5/1X,9HMATERIALS,12X,I5)
  110 FORMAT (I10,2F10.3)
  111 FORMAT (2I5)
  112 FORMAT (E10.3)
  113 FORMAT (I5,5F10.0)
  114 FORMAT (3F10.0/I5,2F10.0)
  115 FORMAT (2I5,2F10.0,I5)
  116 FORMAT (I5,7X,3(F10.1,4X),F5.3,7X,F8.6//)
  117 FORMAT (I2,10A7)
  118 FORMAT (1H1,I2,10A7)
  119 FORMAT (14H0 NODAL POINTS/17X,1HX,10X,1HY)
  120 FORMAT (21H0 BOUNDARY CONDITIONS)
  121 FORMAT (16H0PRINTING SCHEME/5X,22H1. REPORT OUTPUT EVERY,F6.2,2X,4
     1HMSEC/5X,22H2. TERMINATE OUTPUT AT,F6.2,2X,4HMSEC)
  122 FORMAT (1H0,20H MATERIAL PROPERTIES/1X,8HMAT. NO.,7X,2HE1,12X,2HE2
     1,11X,3HG12,10X,3HV12,10X,3HRHO/)
  123 FORMAT (19H0BEAM THICKNESS(IN),7X,F6.3/1X,14HBEAM WIDTH(IN),11X,F6
     1.3/1X,22HSPHERE DENSITY(SL/IN3),4X,F8.6/1X,17HSPHERE RADIUS(IN),8X
     2,F6.3//1X,11HIMPACT NODE,14X,I2/1X,23HIMPACT VELOCITY(IN/SEC),2X,F
     36.1/1X, 39HINTEGRATION TIME INCREMENT( X E-06 SEC),E10.3)
C
      END
      SUBROUTINE CMPD
      COMMON /CONTR/ TITLE(10),NP,NE,NB,NDF,NCN,NLD,NMAT,NSZF,LI,NT4,NDI
     1N,MATP,NPROB
      COMMON /TIME/ T,DT,DDT,TAU,KCON,KCNT
      COMMON /DISP/ Q1,Q2,Q3,Q10,Q20,Q30
      COMMON /DIMB/ TB,WB,PB,NQ,D11
      COMMON CORD(100,2),NOP(200,4),IMAT(200),ORT(25,5),NBC(25),NFIX(25)
     1,R1(200),R2(200),R3(200),R10(200),R20(200),R30(200),FORS(200),SM(2
     200,15),SK(200,15),ISP(200,15),SMPEM(200,15),ESTIF(12,12),EMASS(12,
     312),NFIXK(25)
      COMMON /COMP/ QBR(3,3,25),ABD(6,6),TH(25),ZK(25),MLAYER
      COMMON X1,X2,ND1,ND2
```

```
      REAL IB
      DIMENSION Q(3,3), TK(25)
      DO 102 J=1,3
      DO 102 K=1,3
         ABD(J+3,K+3)=0.0
         ABD(J+3,K)=ABD(J+3,K+3)
         ABD(J,K+3)=ABD(J+3,K)
         ABD(J,K)=ABD(J,K+3)
         DO 101 I=1,25
            QBR(J,K,I)=0.
 101     CONTINUE
 102  CONTINUE
      READ (5,108) MLAYER
      M=MLAYER
      READ (5,109) (L,TH(L),TK(L),I=1,M)
      TTK=0.0
      ZK(1)=TTK
      DO 103 I=1,M
         TTK=TTK+TK(I)
         ZK(I+1)=TK(I)+ZK(I)
 103  CONTINUE
      MM=M+1
      DO 104 I=1,MM
         ZK(I)=ZK(I)-TTK/2.
 104  CONTINUE
      DEL=4.*ATAN(1.)/180.
      DEN=1.-ORT(1,2)*ORT(1,4)**2/ORT(1,1)
      Q(1,1)=ORT(1,1)/DEN
      Q(2,2)=ORT(1,2)/DEN
      Q(2,1)=ORT(1,4)*Q(2,2)
      Q(1,2)=Q(2,1)
      Q(3,3)=ORT(1,3)
      Q(3,2)=0.0
      Q(3,1)=Q(3,2)
      Q(2,3)=Q(3,1)
      Q(1,3)=Q(2,3)
      DO 105 I=1,M
         ANGL=TH(I)*DEL
         C=COS(ANGL)
         W=SIN(ANGL)
         QBR(1,1,I)=Q(1,1)*C**4+2.*(Q(1,2)+2.*Q(3,3))*(C*W)**2+Q(2,2)*W*
     1   *4
         QBR(2,1,I)=(Q(1,1)+Q(2,2)-4.*Q(3,3))*(C*W)**2+Q(1,2)*(W**4+C**4
     1   )
         QBR(1,2,I)=QBR(2,1,I)
         QBR(2,2,I)=Q(1,1)*W**4+2.*(Q(1,2)+2.*Q(3,3))*(C*W)**2+Q(2,2)*C*
     1   *4
         QBR(3,1,I)=(Q(1,1)-Q(1,2)-2.*Q(3,3))*W*C**3+(Q(1,2)-Q(2,2)+(2.)
     1   *Q(3,3))*(W)*(C**3)
         QBR(1,3,I)=QBR(3,1,I)
         QBR(3,2,I)=(Q(1,1)-Q(1,2)-2.*Q(3,3))*W**3*C+(Q(1,2)-Q(2,2)+2.*Q
     1   (3,3))*W*C**3
         QBR(2,3,I)=QBR(3,2,I)
         QBR(3,3,I)=(Q(1,1)+Q(2,2)-2.*Q(1,2)-2.*Q(3,3))*(W*C)**2+Q(3,3)*
     1   (W**4+C**4)
C
C     WRITE(6,500) I,TH(I),TK(I)
C     WRITE(6,510)
C     WRITE(6,520) ((QBR(J,K,I),K=1,3),J=1,3)
C
 105  CONTINUE
      DO 107 J=1,3
      DO 107 K=1,3
         DO 106 I=1,M
            ABD(J,K)=ABD(J,K)+QBR(J,K,I)*(ZK(I+1)-ZK(I))
            ABD(J,K+3)=ABD(J+3,K)+QBR(J,K,I)*(ZK(I+1)**2-ZK(I)**2)/2.
            ABD(J+3,K)=ABD(J,K+3)
            ABD(J+3,K+3)=ABD(J+3,K+3)+QBR(J,K,I)*(ZK(I+1)**3-ZK(I)**3)/3
     1      .
 106     CONTINUE
```

```
  107 CONTINUE
      WRITE (6,111) ((ABD(I,J),J=1,6),I=1,6)
      WRITE (6,110)
C
C     500 FORMAT(2X,*LAYER=*,I2,5X,*ANGLE=*,F5.2,5X,*THICKNESS=*,F7.3)
C     510 FORMAT(2X,*QBAR-MATRIX*)
C     520 FORMAT(5X,3E12.3/)
C
      RETURN
C
  108 FORMAT (I5)
  109 FORMAT (I5,F5.0,F10.0)
  110 FORMAT (///,1X,10HABD MATRIX)
  111 FORMAT (1X,6E11.3)
C
      END
      SUBROUTINE TMX
      COMMON /CONTR/ TITLE(10),NP,NE,NB,NDF,NCN,NLD,NMAT,NSZF,LI,NT4,NDI
     1N,MATP,NPROB
      COMMON /TIME/ T,DT,DDT,TAU,KCON,KCNT
      COMMON /DISP/ Q1,Q2,Q3,Q10,Q20,Q30
      COMMON /DIMB/ TB,WB,PB,NQ,D11
      COMMON /SPHERE/ STF,R,CABV(10),QKONST(10)
      COMMON /PLASTIC/ DISPEM,NDISPEM,FORSPM,DISPM
      COMMON CORD(100,2),NOP(200,4),IMAT(200),ORT(25,5),NBC(25),NFIX(25)
     1,R1(200),R2(200),R3(200),R10(200),R20(200),R30(200),FORS(200),SM(2
     200,15),SK(200,15),ISP(200,15),SMPEM(200,15),ESTIF(12,12),EMASS(12,
     312),NFIXK(25)
      COMMON /COMP/ QBR(3,3,25),ABD(6,6),TH(25),ZK(25),MLAYER
      COMMON X1,X2,ND1,ND2
      REAL IB
C
C       EQUIVALENT MODEL FOR ESTIMATING TIME INTERVAL
C
      IF (STF.NE.0.0) GO TO 101
      STFI=(4./3.)*SQRT(R)/((1.-ORT(NMAT,4)**2)/ORT(NMAT,1)+(1.-ORT(1,4)
     1**2)/ORT(1,1))
      STFA=(4./3.)*SQRT(R)/((1.-ORT(NMAT,4)**2)/ORT(NMAT,1)+1./ORT(1,2))
      STF=STFI
      IF (MATP.EQ.1) STF=STFA
  101 PAI=4.*ATAN(1.)
      BALLM=(4./3.)*PAI*(R**3)*PB
      BL=X2-X1
      AB=WB*TB
      IB=WB*TB**3/12.
      WATP=FLOAT(MATP)
      D11=ORT(1,1)*IB
      IF (MATP.EQ.1) D11=ABD(4,4)
      WRITE (6,105) BL,STF,WATP,D11,ABD(4,4)
      WN1=((D11*(PAI/BL)**4)/(AB*ORT(1,5)))**0.5
      TN1=2.*PAI/WN1
      TD=0.0
      DO 103 I=1,1000
         N=0
         TD=TD+1.0E-7
         F1=0.0
         G1=0.0
         DO 102 J=1,100
            N=N+1
            C=FLOAT(NQ-1)/FLOAT(ND2-ND1)
            WX=SIN(N*PAI*C)
            PP=1./(4.*N**4*TD**2-TN1**2)
            QQ=TN1/(2.*N**2*TD)
            SS=WN1*N**2*TD/2.
            SUMF=(PP*N**2*(1-QQ*SIN(SS))*WX)**2
            SUMG=(PP*COS(SS)*WX)**2
            F1=F1+SUMF
            G1=G1+SUMG
  102    CONTINUE
         SUM1=2.*F1*16.*BL**3*TD**2/(D11*PAI**2)
```

```
      SUM2=2.*G1*16.*ORT(1,5)*AB*BL**7/(D11**2*PAI**4)
      EMT=1./(SUM1+SUM2)
      STFE=1./EMT+1./BALLM
      STFT=STF*STFE
      APHAMAX=(1.25*Q2**2/STFT)**0.4
      TOTALT=2.94*APHAMAX/Q2
      FMAX=STF*APHAMAX**1.5
      EPS=TOTALT-TD
      IF (ABS(EPS).LE.1.E-7) GO TO 104
  103 CONTINUE
  104 WRITE (6,106) TOTALT,TD,FMAX,APHAMAX,EMT
      RETURN
C
  105 FORMAT (//,5X,  5HBEAM=,E10.3,5X,  4HSTF=,E10.3,5X,  5HMATP=,E10.3
     1,5X,  4HD11=,E10.3,5X,  9HABD(4,4)=,E10.3)
  106 FORMAT (//,5X,  7HTOTALT=,E10.3,5X,  3HTD=,E10.3,5X,  5HFMAX=,E10.
     13,5X,  8HAPHAMAX=,E10.3,5X,  4HEMT=,E10.3)
C
      END
```

INTERIM REPORT DISTRIBUTION LIST

NSG3185

CONTACT LAW AND IMPACT RESPONSES OF LAMINATED COMPOSITES

Advanced Research Projects Agency
Washington DC 20525
Attn: Library

Advanced Technology Center, Inc.
LTV Aerospace Corporation
P.O. Box 6144
Dallas, TX 75222
Attn: D. H. Petersen
W. J. Renton

Air Force Flight Dynamics Laboratory
Wright-Patterson Air Force Base, OH 45433
Attn: E. E. Baily
G. P. Sendeckyj (FBC)
R. S. Sandhu

Air Force Materials Laboratory
Wright-Patterson Air Force Base, OH 45433
Attn: H. S. Schwartz (LN)
T. J. Reinhart (MBC)
G. P. Peterson (LC)
E. J. Morrisey (LAE)
S. W. Tsai (MBM)
N. J. Pagano
J. M. Whitney (MBM)

Air Force Office of Scientific Research
Washington DC 20333
Attn: J. F. Masi (SREP)

Air Force Office of Scientific Research
1400 Wilson Blvd.
Arlington, VA 22209

AFOSR/NA
Bolling AFB, DC 20332
Attn: J. Morgan

Air Force Rocket Propulsion Laboratory
Edwards, CA 93523
Attn: Library

Babcock & Wilcox Company
Advanced Composites Department
P.O. Box 419
Alliance, Ohio 44601
Attn: P. M. Leopold

Bell Helicopter Company
P.O. Box 482
Ft. Worth, TX 76101
Attn: H. Zinberg

The Boeing Company
P. O. Box 3999
Seattle, WA 98124
Attn: J. T. Hoggatt, MS. 88-33
T. R. Porter

The Boeing Company
Vertol Division
Morton, PA 19070
Attn: E. C. Durchlaub

Battelle Memorial Institute
Columbus Laboratories
505 King Avenue
Columbus, OH 43201
Attn: L. E. Hulbert

Brunswick Corporation
Defense Products Division
P. O. Box 4594
43000 Industrial Avenue
Lincoln, NE 68504
Attn: R. Morse

Celanese Research Company
86 Morris Ave.
Summit, NJ 07901
Attn: H. S. Kliger

Chemical Propulsion Information Agency
Applied Physics Laboratory
8621 Georgia Avenue
Silver Spring, MD 20910
Attn: Library

Commander
Natick Laboratories
U. S. Army
Natick, MA 01762
Attn: Library

Commander
Naval Air Systems Command
U. S. Navy Department
Washington DC 20360
Attn: M. Stander, AIR-43032D

Commander
Naval Ordnance Systems Command
U.S. Navy Department
Washington DC 20360
Attn: B. Drimmer, ORD-033
M. Kinna, ORD-033A

Cornell University
Dept. Theoretical & Applied Mech.
Thurston Hall
Ithaca, NY 14853
Attn: S. L. Phoenix

Defense Metals Information Center
Battelle Memorial Institute
Columbus Laboratories
505 King Avenue
Columbus, OH 43201

Department of the Army
U.S. Army Aviation Materials Laboratory
Ft. Eustis, VA 23604
Attn: I. E. Figge, Sr.
Library

Department of the Army
U.S. Army Aviation Systems Command
P.O. Box 209
St. Louis, MO 63166
Attn: R. Vollmer, AMSAV-A-UE

Department of the Army
Plastics Technical Evaluation Center
Picatinny Arsenal
Dover, NJ 07801
Attn: H. E. Pebly, Jr.

Department of the Army
Watervliet Arsenal
Watervliet, NY 12189
Attn: G. D'Andrea

Department of the Army
Watertown Arsenal
Watertown, MA 02172
Attn: A. Thomas

Department of the Army
Redstone Arsenal
Huntsville, AL 35809
Attn: R. J. Thompson, AMSMI-RSS

Department of the Navy
Naval Ordnance Laboratory
White Oak
Silver Spring, MD 20910
Attn: R. Simon

Department of the Navy
U.S. Naval Ship R&D Laboratory
Annapolis, MD 21402
Attn: C. Hersner, Code 2724

Director
Deep Submergence Systems Project
6900 Wisconsin Avenue
Washington DC 20015
Attn: H. Bernstein, DSSP-221

Director
Naval Research Laboratory
Washington DC 20390
Attn: Code 8430
I. Wolock, Code 8433

Drexel University
32nd and Chestnut Streets
Philadelphia, PA 19104
Attn: P. C. Chou

E. I. DuPont DeNemours & Co.
DuPont Experimental Station
Wilmington, DE 19898
Attn: D. L. G. Sturgeon

Fiber Science, Inc.
245 East 157 Street
Gardena, CA 90248
Attn: E. Dunahoo

General Dynamics
P.O. Box 748
Ft. Worth, TX 76100
Attn: D. J. Wilkins
Library

General Dynamics/Convair
P.O. Box 1128
San Diego, CA 92112
Attn: J. L. Christian

General Electric Co.
Evendale, OH 45215
Attn: C. Stotler
R. Ravenhall
R. Stabrylla

General Motors Corporation
Detroit Diesel-Allison Division
Indianapolis, IN 46244
Attn: M. Herman

Georgia Institute of Technology
School of Aerospace Engineering
Atlanta, GA 30332
Attn: L. W. Rehfield

Grumman Aerospace Corporation
Bethpage, Long Island, NY 11714
Attn: S. Dastin
J. B. Whiteside

Hamilton Standard Division
United Aircraft Corporation
Windsor Locks, CT 06096
Attn: W. A. Percival

Hercules, Inc.
Allegheny Ballistics Laboratory
P. O. Box 210
Cumberland, MD 21053
Attn: A. A. Vicario

Hughes Aircraft Company
Culver City, CA 90230
Attn: A. Knoell

Illinois Institute of Technology
10 West 32 Street
Chicago, IL 60616
Attn: L. J. Broutman

IIT Research Institute
10 West 35 Street
Chicago, IL 60616
Attn: I. M. Daniel

Jet Propulsion Laboratory
4800 Oak Grove Drive
Pasadena, CA 91103
Attn: Library

Lawrence Livermore Laboratory
P.O. Box 808, L-421
Livermore, CA 94550
Attn: T. T. Chiao
E. M. Wu

Lehigh University
Institute of Fracture &
Solid Mechanics
Bethlehem, PA 18015
Attn: G. C. Sih

Lockheed-Georgia Co.
Advanced Composites Information Center
Dept. 72-14, Zone 402
Marietta, GA 30060
Attn: T. M. Hsu

Lockheed Missiles and Space Co.
P.O. Box 504
Sunnyvale, CA 94087
Attn: R. W. Fenn

Lockheed-California
Burbank, CA 91503
Attn: J. T. Ryder
K. N. Lauraitis
J. C. Ekvall

McDonnell Douglas Aircraft Corporation
P.O. Box 516
Lambert Field, MS 63166
Attn: J. C. Watson

McDonnell Douglas Aircraft Corporation
3855 Lakewood Blvd.
Long Beach, CA 90810
Attn: L. B. Greszczuk

Material Sciences Corporation
1777 Walton Road
Blue Bell, PA 19422
Attn: B. W. Rosen

Massachusetts Institute of Technology
Cambridge, MA 02139
Attn: F. J. McGarry
J. F. Mandell
J. W. Mar

NASA-Ames Research Center
Moffett Field, CA 94035
Attn: Dr. J. Parker
Library

NASA-Flight Research Center
P.O. Box 273
Edwards, CA 93523
Attn: Library

NASA-George C. Marshall Space Flight Center
Huntsville, AL 35812
Attn: C. E. Cataldo, S&E-ASTN-MX
Library

NASA-Goddard Space Flight Center
Greenbelt, MD 20771
Attn: Library

NASA-Langley Research Center
Hampton, VA 23365
Attn: J. H. Starnes
R. A. Pride, MS 188a
M. C. Card
J. R. Davidson

NASA-Lewis Research Center
21000 Brookpark Road
Cleveland, Ohio 44135
Attn: Administration & Technical Service Section
Tech. Report Control, MS. 5-5
Tech. Utilization, MS 3-19
AFSC Liaison, MS. 501-3
Rel. and Quality Assur., MS 500-211
L. Berke, MS 49-3
C. P. Blankenship, MS 105-1
R. F. Lark, MS 49-3
J. C. Freche, MS 49-1
R. H. Johns, MS 49-3
C. C. Chamis, MS 49-3 (10 copies)
T. T. Serafini, MS 49-1
Library, MS 60-3 (2 copies)

NASA-Lyndon B. Johnson Space Center
Houston, TX 77001
Attn: S. Glorioso, SMD-ES52
Library

NASA Scientific and Tech. Information Facility
P.O. Box 8757
Balt/Wash International Airport, MD 21240
Attn: Acquisitions Branch (10 copies)

National Aeronautics & SpaceAdministration
Office of Advanced Research & Technology
Washington DC 20546
Attn: L. Harris, Code RWS
M. Greenfield, Code RWS

National Aeronautics & Space Administration
Office of Technology Utilization
Washington DC 20546

National Bureau of Standards
Eng. Mech. Section
Washington DC 20234
Attn: R. Mitchell

National Technology Information Service
Springfield, VA 22161 (6 copies)

National Science Foundation
Engineering Division
1800 G. Street, NW
Washington DC 20540
Attn: Library

Northrop Corporation Aircraft Group
3901 West Broadway
Hawthorne, CA 90250
Attn: R. M. Verette
G. C. Grimes

Pratt & Whitney Aircraft
East Hartford, CT 06108
Attn: J. M. Woodward

Rockwell International
Los Angeles Division
International Airport
Los Angles, CA 90009
Attn: L. M. Lackman
D. Y. Konishi

Sikorsky Aircraft Division
United Aircraft Corporation
Stratford, CT 06602
Attn: Library

Southern Methodist University
Dallas, TX 75275
Attn: R. M. Jones

Southwest Research Institute
8500 Culebra Road
San Antonio, TX 78284
Attn: P. H. Francis

Space & Missile Systems Organization
Air Force Unit Post Office
Los Angeles, CA 90045
Attn: Technical Data Center

Structural Composites Industries, Inc.
6344 N. Irwindale Avenue
Azusa, CA 91702
Attn: R. Gordon

Texas A&M
Mechanics & Materials Research Center
College Station, TX 77843
Attn: R. A. Schapery

TRW, Inc.
23555 Euclid Avenue
Cleveland, OH 44117
Attn: I. J. Toth

Union Carbide Corporation
P. O. Box 6116
Cleveland, OH 44101
Attn: J. C. Bowman

United Technologies Research Center
East Hartford, CT 06108
Attn: R. C. Novak
Dr. A. Dennis

University of Dayton Research Institute
Dayton, OH 45409
Attn: R. W. Kim

University of Delaware
Mechanical & Aerospace Engineering
Newark, DE 19711
Attn: B. R. Pipes

University of Illinois
Department of Theoretical & Applied Mechanics
Urbana, IL 61801
Attn: S. S. Wang

University of Oklahoma
School of Aerospace Mechanical & Nuclear Engineering
Norman, OK 73069
Attn: C. W. Bert

University of Wyoming
College of Engineering
University Station Box 3295
Laramie, WY 82071
Attn: D. F. Adams

U. S. Army Materials & Mechanics Research Center
Watertown Arsenal
Watertown, MA 02172
Attn: E. M. Lenoe
D. W. Oplinger

V.P. I. and S. U.
Dept. of Eng. Mech.
Blacksburg, VA 24061
Attn: R. H. Heller
       H. J. Brinson
       C. T. Herakovich